OPE'S FOR THE SUPERSTRING IN $AdS_5 \times S^5$ BACKGROUND

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Abstract

Superstrings in $AdS_5 \times S^5$ background can be described through currents in the $psu(2,2|4)$ algebra. Since those currents are not (anti) holomorphic, their OPE's is not determined by symmetry principles and its computation should be performed perturbatively. Using the pure spinor sigma model for this background, we perform this computation in one-loop order.

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1. Introduction

It is not possible to describe the superstring in the $AdS_5 \times S^5$ background using the Ramond-Neveu-Schwarz formalism because of the Ramond-Ramond flux. On the other hand, using the covariant Green-Schwarz formalism it is known how to describe this background [1], although quantization can only be performed using the light-cone gauge, therefore missing the manifest $PSU(2,2|4)$ covariance. Fortunately, there exist a suitable way to covariantly quantize the superstring in this background, which is the pure spinor formalism [2]

In a flat target-space the pure spinor action for the superstring is quadratic, so the OPE's of the worldsheet fields are easly determined and can be used to compute scattering amplitudes. On the other hand, when considering the $AdS_5 \times S^5$ background, the action is written in terms of left invariant currents under $PSU(2,2|4)$ transformations which are neither holomorphic nor antiholomorphic. For that reason it is difficult to compute their OPE's. One approach to this problem is to compute them perturbatively. A first step in this direction was given in [4] [5] , where the OPE's of the worldsheet currents is computed at tree level. The aim of this work is to compute this OPE's perturbatively at the one-loop level. As in the flat target-space, the answer could be useful for computting scattering amplitudes in this background.

The structure of this poster is as follows. In section two review the pure spinor superstring in $AdS_5 \times S^5$ background. In section three we perform a perturbative expansion of the action. In section four we compute some one-loop contributions to the OPE's.

2. Review of the Pure Spinor Superstring in $AdS_5 \times S^5$

As was shown for the first time in [1] , the superstring in $AdS_5 \times S^5$ background can be described using some currents defined in the superalgebra $psu(2,2|4)$. Those currents, which are defined in a left-invariant way, are given by $J^A = (g^{-1}\partial g)^A$, $\overline{J}^A = (g^{-1}\overline{\partial} g)^A$ for g an element in the coset supergroup $PSU(2,2|4)/SO(4,1) \times SO(5)$. The index A denotes $(\underline{a}, \alpha, \hat{\alpha}, [\underline{ab}])$ and $a = 0, \ldots 4$, $a' = 5, \ldots 9$, $\alpha = 1, \ldots 16$, $\hat{\alpha} = 1, \ldots 16$ and $\underline{a}$ denotes both a and a'. In the pure spinor formalism there is also a description in terms of these current, given by the action [2] [6]

$$\begin{aligned} S = & \frac{1}{\alpha^2} \int d^2 z (\frac{1}{2} J^{\underline{a}} \overline{J}^{\underline{b}} \eta_{\underline{ab}} + \delta_{\alpha\hat{\beta}} (J^\alpha \overline{J}^{\hat{\beta}} - 3 J^{\hat{\beta}} \overline{J}^\alpha) \\ & + N_{\underline{ab}} \overline{J}^{[\underline{ab}]} + \hat{N}_{\underline{ab}} J^{[\underline{ab}]} - N_{ab} \hat{N}^{ab} + N_{a'b'} \hat{N}^{a'b'}) + S_\lambda + S_{\widehat{\lambda}}, \end{aligned} \tag{1}$$

where $\delta_{\alpha\hat{\beta}} = (\gamma^{01234})^{\alpha\hat{\beta}}$. $N^{\underline{ab}} = \frac{1}{2}(\lambda \gamma^{\underline{ab}} \omega)$ and $\hat{N}^{\underline{ab}} = \frac{1}{2}(\widehat{\lambda} \gamma^{\underline{ab}} \widehat{\omega})$ are the Lorentz currents for the pure spinors and S_λ, $S_{\widehat{\lambda}}$ are free field actions for the pure spinors λ^α, $\widehat{\lambda}^{\hat{\alpha}}$ respectively. The pure spinor condition means that they satisfy $\lambda^\alpha \gamma^{\underline{a}}_{\alpha\beta} \lambda^\beta = 0$ and $\widehat{\lambda}^{\hat{\alpha}} \gamma^{\underline{a}}_{\hat{\alpha}\hat{\beta}} \widehat{\lambda}^{\hat{\beta}} = 0$. α is a coupling constant of the order $1/R$ where R is the AdS radius in $2\pi\alpha' = 1$ units. Because of their definition, $(J^A, \overline{J}^A)$ satisfy the Maurer-Cartan identities $\partial \overline{J}^A - \overline{\partial} J^A + [J, \overline{J}]^A = 0$, so by making a variation of the action and using those identities, we can find the equations of motion

$$\nabla\overline{J}_2 = -[J_3, \overline{J}_3] - \frac{1}{2}[N, \overline{J}_2] + \frac{1}{2}[J_2, \bar{N}], \tag{2}$$

$$\bar{\nabla} J_2 = [J_1, \overline{J}_1] + \frac{1}{2}[J_2, \bar{N}] - \frac{1}{2}[N, \overline{J}_2] \tag{3}$$

$$\overline{\nabla} J_1 = \frac{1}{2}[N, \overline{J}_1] - \frac{1}{2}[J_1, \bar{N}], \tag{4}$$

$$\nabla\overline{J}_1 = -[J_2, \overline{J}_3] - [J_3, \overline{J}_2] + \frac{1}{2}[N, \overline{J}_1] - \frac{1}{2}[J_1, \bar{N}] \tag{5}$$

$$\nabla\overline{J}_3 = \frac{1}{2}[N, \overline{J}_3] - \frac{1}{2}[J_3, \bar{N}], \tag{6}$$

$$\overline{\nabla} J_3 = [J_2, \overline{J}_1] + [J_1, \overline{J}_2] + \frac{1}{2}[N, \overline{J}_3] - \frac{1}{2}[J_3, \bar{N}], \tag{7}$$

where $\nabla = \partial + [J_0, \,]$ and $\overline{\nabla} = \overline{\partial} + [\overline{J}_0, \,]$. We have supressed the index A and introduced a sub-index $0, 1, 2, 3$ for the currents. This notation stands for $J_0 = J^{[\underline{ab}]} M_{\underline{ab}}$, $J_1 = J^{\alpha} Q_{\alpha}$, $J_2 = J^{\underline{a}} P_{\underline{a}}$, $J_3 = J^{\hat{\alpha}} \hat{Q}_{\hat{\alpha}}$ and similarly for the $\overline{J}$ currents. That is, we have written the currents in terms of the generators of $psu(2,2|4)$, whose structure constants different from zero are

$$f_{\alpha\beta}^{\underline{c}} = 2\gamma_{\alpha\beta}^{\underline{c}}, \quad f_{\hat{\alpha}\hat{\beta}}^{\underline{c}} = 2\gamma_{\hat{\alpha}\hat{\beta}}^{\underline{c}} \tag{8}$$

$$f_{\alpha\hat{\beta}}^{[ef]} = f_{\hat{\beta}\alpha}^{[ef]} = (\gamma^{ef})_{\alpha}{}^{\gamma} \delta_{\gamma\hat{\beta}}, \quad f_{\alpha\hat{\beta}}^{[e'f']} = f_{\hat{\beta}\alpha}^{[e'f']} = -(\gamma^{e'f'})_{\alpha}{}^{\gamma} \delta_{\gamma\hat{\beta}}, \tag{9}$$

$$f_{\alpha\underline{c}}^{\hat{\beta}} = -f_{\underline{c}\alpha}^{\hat{\beta}} = \frac{1}{2}(\gamma_{\underline{c}})_{\alpha\beta}\delta^{\beta\hat{\beta}}, \quad f_{\hat{\alpha}\underline{c}}^{\beta} = -f_{\underline{c}\hat{\alpha}}^{\beta} = -\frac{1}{2}(\gamma_{\underline{c}})_{\hat{\alpha}\hat{\beta}}\delta^{\beta\hat{\beta}}, \tag{10}$$

$$f_{cd}^{[ef]} = \frac{1}{2}\delta_c^{[e}\delta_d^{f]}, \quad f_{c'd'}^{[e'f']} = -\frac{1}{2}\delta_{c'}^{[e'}\delta_{d'}^{f']}, \tag{11}$$

$$f_{[\underline{cd}][\underline{ef}]}^{[\underline{gh}]} = \frac{1}{2}(\eta_{\underline{ce}}\delta_{\underline{d}}^{[\underline{g}}\delta_{\underline{f}}^{\underline{h}]} - \eta_{\underline{cf}}\delta_{\underline{d}}^{[\underline{g}}\delta_{\underline{e}}^{\underline{h}]} + \eta_{\underline{df}}\delta_{\underline{c}}^{[\underline{g}}\delta_{\underline{e}}^{\underline{h}]} - \eta_{\underline{de}}\delta_{\underline{c}}^{[\underline{g}}\delta_{\underline{f}}^{\underline{h}]}), \tag{12}$$

$$f_{[\underline{cd}]\underline{e}}^{\underline{f}} = -f_{\underline{e}[\underline{cd}]}^{\underline{f}} = \eta_{\underline{e}[\underline{c}}\delta_{\underline{d}]}^{\underline{f}}, \quad f_{[\underline{cd}]\alpha}^{\beta} = -f_{\alpha[\underline{cd}]}^{\beta} = \frac{1}{2}(\gamma_{\underline{cd}})_{\alpha}{}^{\beta}, \quad f_{[\underline{cd}]\hat{\alpha}}^{\hat{\beta}} = -f_{\hat{\alpha}[\underline{cd}]}^{\hat{\beta}} = \frac{1}{2}(\gamma_{\underline{cd}})_{\hat{\alpha}}{}^{\hat{\beta}}. \tag{13}$$

This Z_4 gradding for the superalgebra was noted in [7]. The pure spinors have also equations of motion, given by $\overline{\nabla} N = \frac{1}{2}[N, \hat{N}]$ and $\nabla \hat{N} = -\frac{1}{2}[N, \hat{N}]$.

3. Background Field Expansion

We perform a background field expansion as in [7] and [8]. That is, we choose a classical background, given by an element g_0 in the supergroup and parametrize the quantum fluctuations by X as $g = g_0 e^{\alpha X}$. Then, the currents can be written as

$$J = g^{-1}\partial g = e^{-\alpha X} J_0 e^{\alpha X} + e^{-\alpha X}\partial e^{\alpha X}, \tag{14}$$

$$\overline{J} = g^{-1}\overline{\partial} g = e^{-\alpha X} \overline{J}_0 e^{\alpha X} + e^{-\alpha X}\overline{\partial} e^{\alpha X}. \tag{15}$$

The exponentials in (14) can be expanded, giving rise to expressions involving commutators, which can be evaluated by using the structure constants of the $psu(2,2|4)$ Lie superalgebra (8) that is,

$$J = J_0+\alpha(\partial X+[J_0, X])+\frac{\alpha^2}{2}([\partial X, X]+[[J, X], X])+\frac{\alpha^3}{3!}([[\partial X, X], X]+[[[J_0, X], X], X])+..., \quad (16)$$

and similarly for $\overline{J}$. In the last expression J_0 denotes the classical part of J and not the index of the Z_4 gradding. In the following, the expansion of the terms in the action 1 will be written up to cubic terms in the quantum fields, since this is the order relevant for the one-loop computation of the current's OPE'S. We will write only the terms including the quantum fluctuations, since the first term is always a completly classical piece.

Propagators

After performing the expansion up to third order in the quantum fields, the action can be written as a classical piece S_{Cl}, a linear piece in the quantum fields, which is proportional to the equations of motion and therefore will be neglected, a piece from which can be read-off the propagators S_p and another piece containing the second and third order contributions in the quantum fields. The piece from which can be read-off the propagators is

$$S_p = \int d^2z(\frac{1}{2}\partial X^{\underline{a}}\overline{\partial}X^{\underline{b}}\eta_{\underline{ab}} - 4\delta_{\alpha\hat{\beta}}\partial X^{\alpha}\overline{\partial}X^{\hat{\beta}}), \quad (17)$$

therefore,

$$X^a(y)X^b(z) \to -\eta^{ab}ln|y-z|^2, \;\; X^{a'}(y)X^{b'}(z) \to \delta^{a'b'}ln|y-z|^2 \quad (18)$$

$$X^{\alpha}(y)X^{\hat{\beta}}(z) \to -\frac{1}{4}\delta^{\alpha\hat{\beta}}ln|y-z|^2, \quad (19)$$

which in momentum space can be written as

$$X^{\underline{a}}(k)X^{\underline{b}}(l) \to \eta^{\underline{ab}}\frac{\delta^2(k+l)}{|k|^2}, \quad X^{\alpha}(k)X^{\hat{\beta}}(l) \to \frac{1}{4}\delta^{\alpha\hat{\beta}}\frac{\delta^2(k+l)}{|k|^2}, \tag{20}$$

4. One-loop corrections without classical field

From the expression for the expansion of the action, it can be found the following expression containing three quantum fields and non classical field:

$$\begin{aligned} S(X^3) \;=\; & \frac{\alpha^3}{4}[\partial X^{\underline{a}}\overline{\partial}X^{\alpha}X^{\beta}(\gamma_{\underline{a}})_{\alpha\beta} - \partial X^{\underline{a}}\overline{\partial}X^{\hat{\alpha}}X^{\hat{\beta}}(\gamma_{\underline{a}})_{\hat{\alpha}\hat{\beta}} - \overline{\partial}X^{\underline{a}}\partial X^{\alpha}X^{\beta}(\gamma_{\underline{a}})_{\alpha\beta} + \overline{\partial}X^{\underline{a}}\partial X^{\hat{\alpha}}X^{\hat{\beta}}(\gamma_{\underline{a}})_{\hat{\alpha}\hat{\beta}} \\ & +2X^{\underline{a}}\partial X^{\alpha}\overline{\partial}X^{\beta}(\gamma_{\underline{a}})_{\alpha\beta} - 2X^{\underline{a}}\partial X^{\hat{\alpha}}\overline{\partial}X^{\hat{\beta}}(\gamma_{\underline{a}})_{\hat{\alpha}\hat{\beta}}.] \end{aligned} \tag{21}$$

From this equation, an integration by parts allow to find

$$S(X^3) = \frac{\alpha^3}{2}[\partial X^{\underline{a}}\overline{\partial}X^{\alpha}X^{\beta}(\gamma_{\underline{a}})_{\alpha\beta} - \partial X^{\underline{a}}\overline{\partial}X^{\hat{\alpha}}X^{\hat{\beta}}(\gamma_{\underline{a}})_{\hat{\alpha}\hat{\beta}} - \overline{\partial}X^{\underline{a}}\partial X^{\alpha}X^{\beta}(\gamma_{\underline{a}})_{\alpha\beta} + \overline{\partial}X^{\underline{a}}\partial X^{\hat{\alpha}}X^{\hat{\beta}}(\gamma_{\underline{a}})_{\hat{\alpha}\hat{\beta}}] \tag{22}$$

Using this expression we can find that the only OPE's which receive corrections at first order and with no classical fields are

$$J^{\underline{a}}(w)\overline{J}^{\underline{b}}(z) \to 4\alpha^4\eta^{\underline{ab}}[\frac{1}{(w-z)(\bar{w}-\bar{z})} - \delta^{(2)}(w-z)ln|w-z|^2] \tag{23}$$

$$J^{\alpha}(w)\overline{J}^{\hat{\beta}}(z) \to -\frac{5}{4}\alpha^{4}\delta^{\alpha\hat{\beta}}[\frac{1}{(w-z)(\bar{w}-\bar{z})} - \delta^{(2)}(w-z)ln|w-z|^{2}] \tag{24}$$

$$J^{\underline{ab}}(w)\overline{J}^{\underline{cd}}(z) \to -\frac{3}{8}\alpha^{4}\eta^{\underline{a}[\underline{c}}\eta^{\underline{d}]\underline{b}}[\frac{1}{(w-z)(\bar{w}-\bar{z})} - \delta^{(2)}(w-z)ln|w-z|^{2}] \tag{25}$$

$$J^{\underline{ab}}(w)J^{\underline{cd}}(z) \to -\frac{3}{8}\alpha^{4}\eta^{\underline{a}[\underline{c}}\eta^{\underline{d}]\underline{b}}[\frac{1+ln|w-z|^{2}}{(w-z)^{2}}] \tag{26}$$

$$\overline{J}^{\underline{ab}}(w)\overline{J}^{\underline{cd}}(z) \to -\frac{3}{8}\alpha^{4}\eta^{\underline{a}[\underline{c}}\eta^{\underline{d}]\underline{b}}[\frac{1+ln|w-z|^{2}}{(\bar{w}-\bar{z})^{2}}] \tag{27}$$

It is interesting to check that those corrections agree with the equations of motion derived from the action. This aspect is being studied. The contributions to the OPE's involving one classical field are work in progress.

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