Massive modes in AdS_4 / CFT_3 correspondence

Relevant papers: G. Arutynov and S. Frolov, arXiv:0806.4940, D. Bykov,arXiv:0904.0208, P. Sundin, arXiv:0811.2775, K. Zarembo, arXiv:0903.1747

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Abstract:
On this poster we describe some recent work concerning the fate of heavy modes on the string theory side of the AdS_4/CFT_3 correspondence. The correspondence, nowadays dubbed ABJM theory in the literature, relates type IIA strings in a $\mathrm{AdS}_4 \times \mathbb{CP}_3$ background to certain three dimensional Chern Simons theories. Albeit being of a novel nature, the conjecture share many similarities with the well studied AdS_5/CFT_4 holography. Especially, all loop Bethe equations, whose solutions encode the spectrum of the theory, has been proposed. From very general arguments, the fundamental excitations in these equations come in 4+4 solitary Bethe roots. However, as is well known, critical string theory exhibits 8+8 transverse excitations. This leads to the natural question - why this mismatch? A possible explanation to the mismatch is that quantum corrections change the analytic properties of certain Greens functions. The string excitations comes in light and heavy modes and the bare mass of the light modes is exactly half of the heavy ones. It could be so that quantum corrections shift the mass for certain propagators in such a way that its pole disappears. By studying just one of the massive bosonic coordinates, Zarembo showed in arXiv:0903.1747 that this is precisely what happens. This means that the coordinate does not occur as a fundamental excitation in the worldsheet theory of the string. However, there still remains three bosonic and four fermionic massive coordinates. What about these? Will their propagators also exhibit the same change in analytic properties when loop corrections are taken into account? Unfortunately, to be able to answer this question one need to establish the full quartic Hamiltonian. Due to certain technical issues, this Hamiltonian is of a very complicated structure. Nevertheless, equipped with it one can calculate loop corrections to the massive two points function. This is something we currently are working on.

1. Strings on $\mathrm{AdS}_4 \times \mathbb{CP}_3$

The string propagates on the coset manifold

$$\frac{OSP(2,2|6)}{SO(1,3)\times U(3)}, \tag{1}$$

with even part being the isometry group of $\mathrm{AdS}_4\times\mathrm{CP}_3$. The symmetry group exhibits a $\mathbb{Z}_4$ algebra which can be used to construct the string action. Defining,

$$\mathcal{A} = \mathcal{A}^{(0)} + \mathcal{A}^{(2)} + \mathcal{A}^{(1)} + \mathcal{A}^{(3)} = -g^{-1}\,dg, \qquad g \in \mathfrak{osp}(2,2|6) \tag{2}$$

where the superscript denotes grading, one can construct the string Lagrangian as

$$\mathcal{L} = g\,\gamma^{\alpha\beta}\,\mathcal{A}^{(2)}_\alpha\,\mathcal{A}^{(2)}_\beta - g\,\kappa\,\epsilon^{\alpha\beta}\,\mathcal{A}^{(1)}_\alpha\,\mathcal{A}^{(3)}_\beta, \tag{3}$$

where greek indices are worldsheet indices, i.e $\sigma^\alpha = (\tau,\sigma)$.

Symmetries:

- Invariance under global $OSP(2,2|6)$
- Invariant under worldsheet diffeomorphism and Weyl scalings
- Fermionic κ symmetry: 24 $\Rightarrow$ 16 real fermions. .

To fix the bosonic symmetries, it is convenient to introduce, $\Pi = \gamma^{0\alpha}\mathcal{A}^{(2)}_\alpha$, which allows us to rewrite the Lagrangian as

$$\mathcal{L} = g\,\Pi\,\mathcal{A}^{(2)}_0 - g\,\kappa\,\epsilon^{\alpha\beta}\,\mathcal{A}^{(1)}_\alpha\,\mathcal{A}^{(3)}_\beta + \lambda_1(\Pi^2 + (\mathcal{A}^{(2)}_1)^2) + \lambda_2(\Pi\,\mathcal{A}^{(2)}_1), \tag{4}$$

where λ_1 and λ_2 are Lagrange multipliers. This way of writing the Lagrangian allows us to fix a light-cone gauge without having to worry about the worldsheet metric.
Introducing $x^\pm = t \pm \phi$, where t and ϕ denotes time and angle variable in AdS and CP spaces, one imposes

$$x^+ = \tau, \qquad P_+ = 1. \tag{5}$$

In this gauge the Hamiltonian becomes

$$\frac{\delta\mathcal{L}}{\delta\dot{x}^+} = P_- = -\mathcal{H}. \tag{6}$$

This gauge fixing has the consequence that the symmetry gets reduced

$$OSP(2,2|6) \quad\Rightarrow\quad SU(2|2)\times U(1), \tag{7}$$

or, more precisely, only the subalgebra $\mathfrak{su}(2|2)\oplus\mathfrak{u}(1)$ commutes with the light-cone Hamiltonian. As was the case for the $\mathrm{AdS}_5\times\mathrm{S}^5$ case, the off shell symmetry algebra undergoes central extension (see work by Bykov).
After gauge fixing (Weyl, diff and κ), the string has 8_B+8_F degrees of freedom. It is convenient to choose representations that transform covariantly under the bosonic part of the symmetry group $(SU(2)_{AdS}\times SU(2)_{CP_3}\times U(1))$. Denoting the AdS $SU(2)$ with Latin indices and the CP_3 $SU(2)$ with Greek indices, we find the following multiplets:

M=1	**Bosons**:	$\{Z^a_b,\ \ y\}$	**Fermions**:	$\{s^a_\alpha\}$
M=$\frac{1}{2}$	**Bosons**:	$\{\omega^\alpha\}$	**Fermions**:	$\{\kappa^{+,a},\ \ \kappa_{-,a}\}$.

where ω^α and $\kappa^\pm$ are the only fields charged under the $U(1)$. We also expressed the three transverse AdS coordinates in terms of Pauli matrices, $Z^a_b = z_i\,\sigma^a_{i,b}$. To get a feeling for things we can compare it to the more well known $\mathrm{AdS}_5\times\mathrm{S}^5$ case where the bosonic (gauge fixed) isometry group was $SU(2)^4$. There one had two sets of fermionic and bosonic fields where each set transformed in different bifundamental $SU(2)\times SU(2)$ representations. For the case at hand, we can roughly consider the Z^b_a and s^a_β coordinates as 'half of the $\mathrm{AdS}_5\times\mathrm{S}^5$ spectrum'. The other coordinates mix, or contract, in a more complicated way than the AdS_5 case. We will see below that the mixing get's considerably more complicated for the type IIA string.

2. Strong coupling expansion

The Lagrangian (4) is as it stands highly non trivial. To be able to extract any useful information we have to consider various simplifying limits. The one we use is a strong coupling expansion, or equivalently, an expansion in number of fields. This has the effect that the string Lagrangian expands as

$$\mathcal{L} = \mathcal{L}_2 + \frac{1}{\sqrt{g}}\mathcal{L}_3 + \frac{1}{g}\mathcal{L}_4 + ..., \tag{8}$$

where the subscript denotes the number of fields in each expansion term. In canonical form, this equals

$$\frac{1}{2}\mathscr{L} = \\ p_i\,\dot{z}_i + p_y\,\dot{y} + \dot{w}^\alpha\,\bar{p}_\alpha + \dot{\bar{\omega}}_\alpha\,p^\alpha + i\bar{s}_{a\beta}\,\dot{s}^{a\beta} + i\bar{\kappa}^+_a\,\dot{\kappa}^{+,a} + i\bar{\kappa}^{-,a}\,\dot{\kappa}^-_a - \mathcal{H}, \tag{9}$$

where we introduced canonical momenta for the bosonic fields. In the above we have also shifted the fermions, $\chi \Rightarrow \chi + \phi(\chi,x,p)$, so that $\mathcal{H}$ (to quartic order) does not contain any time derivatives of fermionic coordinates. This shift unfortunately complicates the general structure of the interacting part of the Lagrangian quite considerably.
The quadratic Hamiltonian is found to be

$$\begin{aligned}\mathcal{H}_2 = &-p_i^2 - 4\bar{p}_\alpha\,p^\alpha - p_y^2 - \frac{1}{4}\big(y^2 + z_i^2 + \frac{1}{4}\bar{\omega}_\alpha\,\omega^\alpha\big) - \frac{1}{4}\big(z_i'^2 + y'^2 + \bar{\omega}'_\alpha\,\omega'^\alpha\big)\\ &-\bar{s}_{a\beta}\,s^{a\beta} - \frac{1}{2}\big(\bar{\kappa}_{+,a}\,\kappa^{+,a} + \bar{\kappa}^{-,a}\,\kappa_{-,a}\big) - i\kappa\big(\kappa_{-,a}\,\kappa'^{+,a} + \bar{\kappa}_{+,a}\,\bar{\kappa}'^{-,a}\big)\\ &+\frac{i}{2}\kappa\big(s^{a\beta}\,s'_{a\beta} + \bar{s}_{a\beta}\,\bar{s}'^{a\beta}\big),\end{aligned}$$

and the cubic part is

$$\begin{aligned}\sqrt{g}\,\mathcal{H}_3 = &\,\kappa\Big(\kappa_{-,a}\,\bar{\kappa}'^{-,b} - \kappa_{+,a}\,\kappa'^{+,b} + \kappa'_{-,a}\,\bar{\kappa}^{-,b} - \bar{\kappa}'_{+,a}\,\kappa^{+,b}\Big)\,Z^a_b\\ &+2i\Big(\kappa_{-,a}\,\kappa'^{+,b} + \bar{\kappa}_{+,a}\,\bar{\kappa}'^{-,b} - \kappa'_{-,a}\,\kappa^{+,b} - \bar{\kappa}'_{+,a}\,\bar{\kappa}^{-,b}\Big)\,Z'^a_b\\ &+2i\kappa\Big(\bar{\kappa}'_{+,a}\,\kappa^{+,b} - \kappa_{-,a}\,\bar{\kappa}'^{-,b} + \kappa'_{-,a}\,\bar{\kappa}^{-,b} - \bar{\kappa}_{+,a}\,\kappa'^{+,b}\Big)\,Pz^a_b\\ &+2\kappa\Big(\epsilon^{\alpha\beta}\big(\bar{s}_{a\alpha}\,\bar{\kappa}'^{-,a} - \bar{s}'_{a\alpha}\,\bar{\kappa}^{-,a}\big) + \epsilon_{ab}\big(\kappa^{+,a}\,s'^{b\beta} - s^{a\beta}\,\kappa'^{+,b}\big)\Big)\bar{p}_\beta\\ &+2\kappa\Big(\epsilon^{ab}\big(\bar{\kappa}_{+,a}\,\bar{s}'_{b\beta} - \bar{s}_{a\beta}\,\bar{\kappa}'_{+,b}\big) + \epsilon_{\alpha\beta}\big(\kappa_{-,a}\,s'^{a\alpha} - \kappa'_{-,a}\,s^{a\alpha}\big)\Big)p^\beta\\ &-\kappa\frac{i}{4}\Big(\epsilon^{\alpha\beta}\big(\bar{s}_{a\alpha}\,\bar{\kappa}'^{-,a} + \bar{s}'_{a\alpha}\,\bar{\kappa}^{-,a}\big) + \epsilon_{ab}\big(s^{a\beta}\,k'^{+,b} + \kappa^{+,a}\,s'^{b\beta}\big)\Big)\bar{w}_\beta\\ &+\kappa\frac{i}{4}\Big(\epsilon^{ab}\big(\bar{s}_{a\beta}\,\bar{\kappa}'_{+,b} + \bar{\kappa}_{+,a}\,\bar{s}'_{b\beta}\big) - \epsilon_{\alpha\beta}\big(\kappa_{-,a}\,s'^{a\alpha} + k'_{-,a}\,s^{a\alpha}\big)\Big)\,w^\beta\\ &-\Big(\epsilon^{\alpha\beta}\big(\bar{s}_{a\alpha}\,\kappa'^{+,a} - \bar{s}'_{a\alpha}\,\kappa^{+,a}\big) + \epsilon_{ab}\big(\bar{\kappa}^{-a}\,s'^{b\beta} - s^{a\beta}\,\bar{\kappa}'^{-,b}\big)\Big)\bar{w}'_\beta\\ &-\Big(\epsilon^{ab}\big(\kappa_{-,a}\,\bar{s}'_{b\beta} - \bar{s}_{a\beta}\,\kappa'_{-,b}\big) + \epsilon_{\alpha\beta}\big(\bar{\kappa}_{+,a}\,s'^{a\alpha} - \bar{\kappa}'_{+,a}\,s^{a\alpha}\big)\Big)\,w'^\beta\\ &+\frac{i}{2}y\big(\bar{p}_\alpha\,w^\alpha - \bar{w}_\alpha\,p^\alpha\big).\end{aligned}$$

There is of course also a quartic part,

$$g\,\mathcal{H}_4 = \mathcal{H}_{4,BBBB} + \mathcal{H}_{4,BBFF} + \mathcal{H}_{4,FFFF}, \tag{10}$$

in an obvious notation. Unfortunately, $\mathcal{H}_4$ is *horribly* complicated. However, for the calculation we are performing, only relatively simple parts of the quartic piece are needed.
Before discussing the analytic structure of the massive propagators let us point out a few general features of the interacting Hamiltonian. First, and somewhat obvious by now, we see that a novel feature for the type IIA string is that it exhibits interactions already at the cubic level. In contrast to, for example, the $\mathrm{AdS}_5\times\mathrm{S}^5$ case, the relevant scattering processes now get additional contributions. Most evident is that we can get a three vertex loop at order g^{-1}. This has the consequence that we have to consider two distinct loop diagrams for the corrected two point functions. This is indeed also what we will do in the below. However, before that, let us comment on the point particle structure of the above Hamiltonian. By now it is more or less known that the spectrum of type IIA supergravity should be reproduced by sending the string length parameter to zero. Since the supergravity spectrum is fully determined by the free quadratic part one might ask what the meaning of the surviving higher order terms is. For example, already at cubic order we see

$$\mathcal{H}_3|_{\sigma\to 0} = \frac{i}{2}y\big(\bar{p}_\alpha\,w^\alpha - \bar{w}_\alpha\,p^\alpha\big). \tag{11}$$

It so turns out that these can be simply removed by performing a unitary transformation of the quantum Hamiltonian (or, an equivalent canonical transformation on the classical phase space). I.e, for a transformation U, we can send

$$\mathcal{H} \quad\Rightarrow\quad e^{-i\,U}\,\mathcal{H}\,e^{i\,U},$$

in such a way that we add additional terms which precisely cancels the unphysical higher order terms. In fact, one can also perform a unitary transformation for the $\sigma \neq 0$ case in such a way that the cubic terms get shifted to quartic order. This of course comes with the price of additional higher order terms.

3. Fate of the massive modes

We are now in position to start investigating the higher loop corrections to the massive propagators. The bare propagators are given by,

$$\begin{aligned}\langle 0|T\big\{y(\sigma)\,y(\sigma')\big\}|0\rangle &>= -\frac{2i}{(2\pi)^2}\int d^2p\frac{e^{-i\bar{p}\cdot(\bar{\sigma}-\bar{\sigma}')}}{\bar{p}^2 - 1 + i\epsilon},\\ \langle 0|T\big\{z_i(\sigma)\,z_j(\sigma')\big\}|0\rangle &>= -\frac{2i}{(2\pi)^2}\int d^2p\frac{\delta_{ij}}{\bar{p}^2 - 1 + i\epsilon}\,e^{-i\bar{p}\cdot(\bar{\sigma}-\bar{\sigma}')},\\ \langle 0|T\big\{s^{a\alpha}(\sigma)\,\bar{s}_{b\beta}(\sigma')\big\}|0\rangle &= \frac{1}{2}\frac{i}{(2\pi)^2}\int d^2p\frac{(p_0+1)\delta^a_b\,\delta^\alpha_\beta}{\bar{p}^2 - 1 + i\epsilon}e^{-i\bar{p}\cdot(\bar{\sigma}-\bar{\sigma}')}.\end{aligned} \tag{12}$$

To calculate mass corrections we need to consider two distinct type of loop diagrams. Due to the poster package of Latex[1], which for some reason or another, refuse to work with Feynman graphs, we are forced to present them in the form of equations. For two massive fields A and B, we have

$$\begin{aligned}&\langle\Omega|T\big\{A(x)\,B(y)\big\}|\Omega\rangle = \langle 0|T\big\{A(x)\,B(y)\big\}|0\rangle\\ &-\frac{1}{g}\langle 0|T\Big\{A(x)\,B(y)\Big(i\int d^2\sigma\mathcal{H}_4(\sigma) + \frac{1}{2}\int d^2\sigma\,d^2\sigma'\,\mathcal{H}_3(\sigma)\,\mathcal{H}_3(\sigma')\Big)\Big\}|0\rangle.\end{aligned} \tag{13}$$

The first term gives a tadpole diagram and the second a one loop three vertex diagram.
To be a little more illustrative, let's consider the simplest case, $\langle\Omega|\,T\big\{y(\sigma)\,y(\sigma')\big\}|0\rangle$ which was calculated by Zarembo. From very general considerations he argued that only the cubic Hamiltonian is necessary for the calculation. However, nothing is lost by being a tad more careful so let's consider the full contribution,

$$\begin{aligned}\mathcal{H}_y = &\\ &\frac{i}{2\sqrt{g}}\Big(\bar{p}_\alpha\,\omega^\alpha - \bar{\omega}_\alpha\,p^\alpha\Big)y + \frac{1}{g}\Big\{-\frac{1}{2}\Big(\bar{\kappa}_{+,a}\,\kappa^{+,a} - \kappa_{-,a}\,\bar{\kappa}^{-,a}\Big)\big(\frac{1}{4}y'^2 - \frac{1}{4}y^2 + p_y^2\big)\\ &+\frac{1}{2}\bar{s}'_{a\beta}\,s'^{a\beta}\,y^2 - \frac{i}{4}\big(\bar{\kappa}'_{+,a}\,\bar{\kappa}^{-,a} + \kappa'_{-,a}\,\kappa^{+,a} - \kappa_{-,a}\,\kappa'^{+,a} - \bar{\kappa}_{+,a}\bar{\kappa}'^{-,a}\big)y^2\\ &+\frac{3}{4}p_y^2\,y^2 - \frac{7}{128}y^4 + \frac{1}{16}y^2\,y'^2 + \frac{3}{32}\bar{\omega}_\alpha\,\omega^\alpha\,y'^2 + \frac{1}{8}\bar{\omega}'_\alpha\,\omega'^\alpha\,y^2 + 2y^2\,\bar{p}_\alpha\,p^\alpha\\ &-\frac{1}{8}p_y^2\,\bar{\omega}_\alpha\,\omega^\alpha - \frac{1}{128}\bar{\omega}_\alpha\,\omega^\alpha\,y^2 + \frac{1}{2}p_y^2\,p_i^2 - \frac{1}{8}y'^2\,z_i^2 + \frac{1}{8}y^2\,z_i'^2 - \frac{1}{2}p_y^2\,z_i^2\Big\}.\end{aligned} \tag{14}$$

Plugging this into (13) together with some work indeed shows that the pole disappears as advocated from Zarembo's calculation. Of course, the above calculation also give rise to divergent parts. These can readily be isolated through dimensional regularization. Taking into account all massive fields, one should be able to show that these terms cancel among themselves.
As said, this is not something new. What we are in the progress of investigating is the corresponding calculations for the remaining massive fields, Z^a_b and s^a_α. The calculation is straightforward but rather tedious. For example, already for the cubic Hamiltonian one see that things get quite more involved.

4. Summary and outlook

- **We have derived the full quartic Hamiltonian for strings in $\mathrm{AdS}_4\times\mathbb{CP}_3$. This has been done in a fully covariant notation with respect to the bosonic, gauge fixed, isometry group $SU(2)\times SU(2)\times U(1)$.**
- **The resulting Hamiltonian exhibits interactions already at the cubic order of number of fields. It's quartic part is quite involved, mainly due to a fermionic shift that had to be performed to make the Lagrangian canonical.**
- **The string excitations come in light and heavy modes, with $M=1$ and $M=\frac{1}{2}$ respectively. A proposed set of Bethe equations hint that only the light modes constitute fundamental excitations in the scattering theory.**
- **By continuing a line of research initiated by Zarembo we analyse the pole structure of the Green functions of massive excitations. It will probably turn out that the pole in the loop corrected propagators vanish for all massive fields.**
- **Having the full interacting, quartic, theory in a nice notation, a natural continuation of this work is to study the full scattering matrix. Of course, due to the complexity of the theory, this will be quite a involved task. If to involved, various closed subsectors can be considered.**

[1] Or possibly, but of course unlikely, due to user related incompetence.