Correlator of Wilson and ‘t Hooft Loops from AdS/CFT

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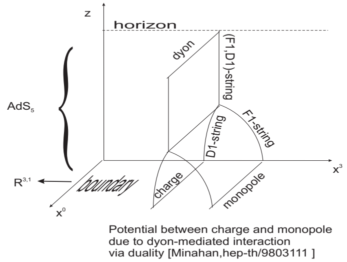
Alexander GORSKY[1,2], Alexander MONIN[3], Andrew V. ZAYAKIN[1,4]

[1] ITEP, Moscow [2] FTPI, Minnesota [3]University of Minnesota [4]LMU-München

Motivation

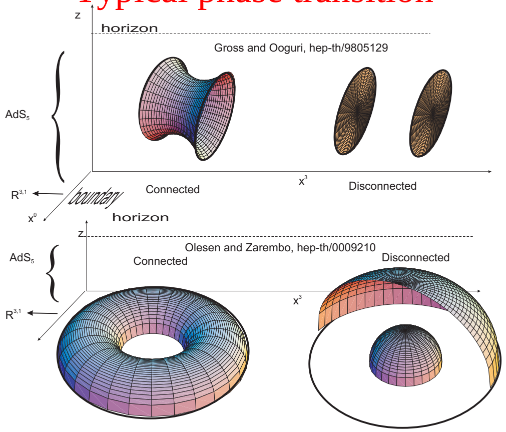
- Monopole-like degrees of freedom are relevant for the description of the quark-gluon plasma.
- It is interesting to look at the the stringy mechanism of interaction.
- For that purpose we study a correlator between a Wilson and a ‘t Hooft loops by AdS/CFT means
- We generalize to a correlator of arbitrary dyons and observe interesting $SL(2,\mathbb{Z})$ modular properties of the correlator

Junction



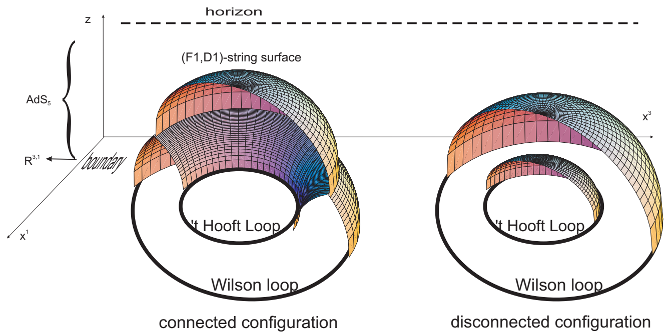
The charge-monopole-dyon junction is a mediator of charge-monopole interaction

Typical phase transition



There exist a lot of phase transition in AdS/CFT correspondence, e.g. Gross--Ooguri phase transition, Olesen--Zarembo shown in the pictures above.

Novel phase transition



We have studied a novel phase transition between the connected (left) and disconnected (right) configurations shown above.

Geometry and Action

We work in the following finite-temperature metric

$$ds^2 = \frac{R^2}{z^2}\left(-h(z)dt^2 + dx_i^2\right) + \frac{R^2}{h(z)}\frac{dz^2}{z^2}$$

with solutions subject to conditions

$$h(z) = \begin{cases} 1 - \frac{z^4}{z_0^4}, & z_0 > 2/3 \\ z^2 + 1 - \mu z^4, & z_0 < 2/3 \end{cases}$$

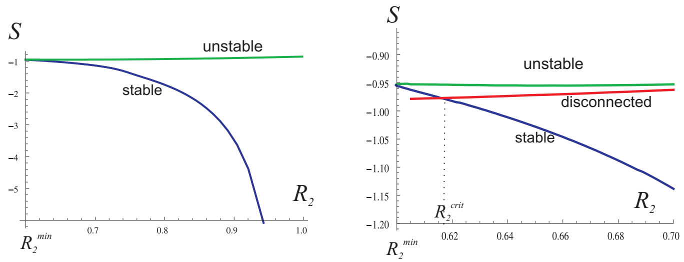
Action on each piece of *(p,q)* string is given as

$$S = T_{p,q}\int dz\, \frac{r}{z^2}\sqrt{\frac{1}{h(z)} + r'^2}.$$

$$\begin{cases} z_1'(0) = 0 \\ z_2(R_2) = 0 \\ z_3(R_1) = 0 \\ z_1(\bar{r}) = z_2(\bar{r}) = z_3(\bar{r}) \\ T_{1,1}\tau_1^\mu(\bar{r}) + T_{0,1}\tau_2^\mu(\bar{r}) + T_{1,0}\tau_3^\mu(\bar{r}) = 0 \end{cases}$$

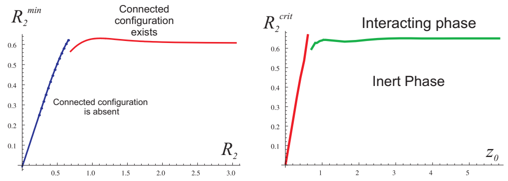
Solving EOM with given conditions we get the dynamics from classical action on EOM.

Connected and Disconnected Configurations



Action on the connected configuration has two branches, stable and unstable one. The stable connected configuration competes in the dynamics with the disconnected one. This causes the system to possess a phase transition.

Interpretation of the String Configurations



Dependent on the ratio of loop radii, different types of dynamics are realized:

$R_2 > R_{min}(z_0)$	Connected configuration exists
$R_2 > R_{crit}(z_0)$	Connected configuration dominates: monopoles and charges are non-perturbatively correlated
$R_2 < R_{crit}(z_0)$	Disconnected configuration dominates, only PT interaction between monopoles and charges

$SL(2,\mathbb{Z})$ Modularity Properties

- The correlator we have considered had (1,0) and (0,1) particles on the boundary
- It is natural to consider (p_1,q_1) and (p_2,q_2) dyons instead.
- The correlator then will live in the representations of the tensor product

$$SL(2,Z)\otimes SL(2,Z)$$

- Those are realized on modular forms of one complex variable *f(z)*, where one expects

$$z = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$$

- We observe the following modularity property of the full action S on charge and monopole:

$$S(z) = zS(-1/z)$$

Conclusions

- We have found a phase transition between correlated and (PT) uncorrelated phases of monopoles and dyons
- The transition means that below some specific ratio of radii of ‘t Hooft and Wilson loops the dual configuration in AdS is a disconnected one, whereas above this ratio – a connected one
- The correlator of general dyon loops possesses interesting modularity properties under $SL(2,\mathbb{Z})$ transformations, constituting a representation of it with regard to $z=\theta/(2\pi)+4\pi i/g^2$ as complex parameter
- In particular, the charge-monopole correlator transforms with weight 1 under this transformation: $S(z)=zS(-1/z)$.

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