

# Anomalous dimensions at four loops in ABJM and ABJ

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02.07.09, IGST, Golm

J. Minahan, O. Ohlsson Sax, C. S., work in progress

# Outline

Introduction and motivation

$SU(2) \times SU(2)$  subsector

Extraction of  $h(\lambda)$

Field theory calculation: 2-loop warm-up

Field theory calculation: 4-loops

From ABJM to ABJ  
Symmetries

What made us suffer

# AdS<sub>4</sub>/CFT<sub>3</sub> (ABJM) correspondence

[Aharony, Bergman, Jafferis, Maldacena]

Type II A ST AdS<sub>4</sub> × CP<sup>3</sup>

⇔

3-dim.  $\mathcal{N} = 6$  CS theory

energy  $E$   
(semicl.) strings

$\lambda \gg 1$

$\lambda \ll 1$

anom. dim.  $\gamma$   
comp. operators

$\mathcal{O}_J \propto \text{tr } \phi_{n_1} \dots \phi_{n_J}$

integrable systems  
(asymptot.) Bethe ansätze

[Minahan, Zarembo]

[Bak, Rey]

[Gromov, Vieira]

magnon dispersion relation

[Beisert, Dippel, Staudacher]

[Beisert]

[Arutyunov, Frolov, Zamaklar]

$$E = \frac{1}{2} \left( \sqrt{1 + 16h(\lambda)^2 \sin^2 \frac{\theta}{2}} - 1 \right)$$

$$h(\lambda)^2 = ?$$

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$$h(\lambda)^2 = \frac{\lambda}{(4\pi)^2}$$

trivial in AdS<sub>5</sub>/CFT<sub>4</sub>

BMN limit  
giant magnons

[Nishioka, Takayanagi]

[Gaiotto, Giombi, Yin]

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$$\gamma = \lambda^2 \gamma_2$$

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## $SU(2) \times SU(2)$ subsector

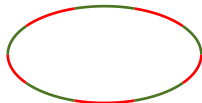
bifundamentals  $(N, \bar{N})$  and  $(\bar{N}, N)$  of  $U(N) \times \overline{U(N)}$  gauge group  
in ABJ:  $(M, \bar{N})$  and  $(\bar{M}, N)$  of  $U(M) \times \overline{U(N)}$

[Aharony, Bergman, Jafferis]

$$Y^A_{ab} = (A_1, A_2, B_1^\dagger, B_2^\dagger), \quad Y_A^{\dagger ba} = (A_1^\dagger, A_2^\dagger, B_1, B_2)$$

gauge invariant composite operator

$$\mathcal{O}_{2L} = \text{tr}(Y^{A_1} Y_{A_2}^\dagger Y^{A_3} Y_{A_4}^\dagger \dots Y^{A_{2L-1}} Y_{A_{2L}}^\dagger) =$$



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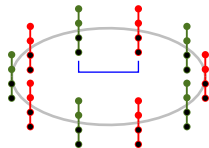
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4 and  $\bar{4}$  of  $SU(4)$ : flavour traces possible

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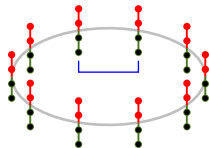
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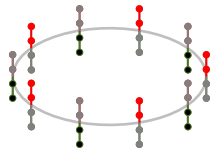
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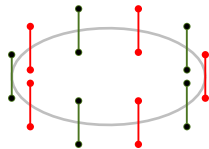
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$SU(2) \times SU(2)$  subsector, free of flavour traces

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# Bethe ansatz

[Minahan, Zarembo]  
[Bak, Rey]  
[Gromov, Vieira]

$SU(2) \times SU(2)$  sector:

two copies of the  $XXX_{\frac{1}{2}}$  Heisenberg spin chain

only coupled via momentum conservation

and BES dressing phase

[Beisert, Eden, Staudacher]

$$\sum_{j=1}^{M_2} p_{2,j} + \sum_{j=1}^{M_2} p_{\dot{2},j} = 0,$$

$$e^{ip_{2,j}L} = \prod_{k \neq j}^{M_2} S(u_{2,k}, u_{2,j}) \prod_{k=1}^{M_2} \sigma(u_{2,k}, u_{\dot{2},j})$$

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dispersion relation:  $E = E_2 + E_{\dot{2}}$

$$E_2 = \sum_{j=1}^{M_2} \frac{1}{2} \left( \sqrt{1 + 16h(\lambda)^2 \sin^2 \frac{p_j}{2}} - 1 \right)$$

$$E_{\dot{2}} = \sum_{j=1}^{M_2} \frac{1}{2} \left( \sqrt{1 + 16h(\lambda)^2 \sin^2 \frac{p_j}{2}} - 1 \right)$$

# States in the $SU(2) \times SU(2)$ subsector

ground state,  $E = 0$

$$\mathcal{O}_{2L,(0,0)} = \text{tr}((A_1 B_1^\dagger)^L) = \text{diag}(\uparrow, \uparrow, \uparrow, \uparrow, \uparrow, \uparrow, \uparrow, \uparrow)$$

single magnon states,  $E = 0$

$$\mathcal{O}_{2L,(1,0)} = \text{tr}(A_2 B_1^\dagger (A_1 B_1^\dagger)^{L-1}) = \text{diag}(\uparrow, \uparrow, \uparrow, \uparrow, \uparrow, \downarrow, \uparrow, \uparrow), \quad p_2 = 0$$

$$\mathcal{O}_{2L,(0,1)} = \text{tr}(A_1 B_2^\dagger (A_1 B_1^\dagger)^{L-1}) = \text{diag}(\uparrow, \uparrow, \uparrow, \uparrow, \downarrow, \uparrow, \uparrow, \uparrow), \quad p_2 = 0$$

two magnon states,  $p_2 = -p_2 = \frac{2\pi n}{L}$ ,  $E = \sqrt{1 + 16h(\lambda)^2 \sin^2 \frac{p_2}{2}} - 1$

$$\mathcal{O}_{2L,(1,1)} = \text{tr}(A_2 B_1^\dagger (A_1 B_1^\dagger)^k A_1 B_2^\dagger (A_1 B_1^\dagger)^{L-k-2}) = \text{diag}(\uparrow, \uparrow, \uparrow, \downarrow, \uparrow, \uparrow, \downarrow, \uparrow)$$

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apply  $D_4$  to single magnon momentum eigenstate

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$$D_4 \psi_p \rightarrow (\delta_{4,0} + \delta_{4,3} \cos p + \delta_{4,5} \cos 2p) \psi_p$$

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$$E_2 = \frac{1}{2} \left( \sqrt{1 + 16h(\lambda)^2 \sin^2 \frac{p}{2}} - 1 \right) \rightarrow 2h_4 - 6 - 2(h_4 - 4) \cos p - 2 \cos 2p$$

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$\Rightarrow$  range three interactions determine  $h_4 = -\frac{1}{2} \delta_{4,3} + 4$

$\Rightarrow$  range five interactions  $\delta_{4,5} = -2$



## Scheme independence of $h(\lambda)$

Bethe ansatz only depends on  $h(\lambda)$ , ABJM theory depends on  $\lambda = \lambda(\mu)$   
dilatation operator from  $\mathcal{O}_a = Z_a^b \mathcal{O}_b^{\text{bare}}$

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scheme change:

$$\lambda \rightarrow \lambda'(\lambda) = \lambda + a\lambda^3 + \dots$$

$$Z'(\mu, \lambda') = U(\lambda) Z(\mu, \lambda) (1\lambda + A\lambda^2 + \dots) U(\lambda)^{-1}$$

in a **conformal** QFT:  $\mu \frac{d}{d\mu} \lambda = \beta(\lambda) = 0$

$$D'(\lambda') = U(\lambda) D(\lambda) U(\lambda)^{-1} + \beta(\lambda) \text{ rest} = U(\lambda) D(\lambda) U(\lambda)^{-1}$$

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$h(\lambda)$  fixed by matching  $E(h(\lambda)) = \gamma(\lambda)$

$\Rightarrow h(\lambda)$  scheme independent

coefficients of perturbative expansions w.r.t.  $\lambda$  and  $\lambda'$  can be different

$$h'(\lambda')^2 = \lambda'^2 + h'_4 \lambda'^4 + \dots = \lambda^2 + h_4 \lambda^4 + \dots = h(\lambda)^2, \quad h'_4 = h_4 - 2a$$

# Regularization and $\epsilon$ -tensors

Dimensional reduction:

consistent up to  $\left\{ \begin{array}{ll} 3 \text{ loops:} & \text{pure CS} \\ 2 \text{ loops:} & \text{CS} + \text{matter} \end{array} \right.$

[Chen, Semenoff, Wu]  
[Chen, Semenoff, Wu]  
[Alves, Gomes, Pinheiro, da Silva]

observation: in all appearing two- and four-loop integrals:

$$\text{number}(\epsilon_{\mu\nu\rho}) + \underbrace{\text{number}(\text{tr}(\gamma^{\mu_1} \dots \gamma^{\mu_{2n-1}}))}_{\sim \epsilon^{\mu_i \mu_j \mu_k}} = 2 \text{ IN}$$

expand product of **even number** of  $\epsilon$ -tensors  $\rightarrow$  scalar products  
reduce dimensions in the integrals to  $D = 3 - 2\epsilon$

# Field theory calculation: 2-loop warm-up

with symmetry factors to avoid overcounting:

$$\text{Diagram} = \frac{\lambda^2}{4} \frac{1}{\epsilon} \frac{1}{4} \left( -2 \text{Diagram}_1 - 2 \text{Diagram}_2 + \text{Diagram}_3 + \text{Diagram}_4 - 2 \text{Diagram}_5 + 4 \text{Diagram}_6 \right)$$

$$\frac{1}{2} \left( \text{Diagram}_7 + \text{Diagram}_8 \right) = -\frac{\lambda^2}{4} \frac{1}{2\epsilon} \left( \text{Diagram}_9 + \text{Diagram}_{10} \right)$$

$$\frac{1}{2} \left( \text{Diagram}_{11} + \text{Diagram}_{12} \right) = +\frac{\lambda^2}{4} \frac{1}{4\epsilon} \left( \text{Diagram}_9 + \text{Diagram}_{10} \right)$$

$$\frac{1}{2} \left( \text{Diagram}_{13} + \text{Diagram}_{14} \right) = +\frac{\lambda^2}{4} \frac{1}{4\epsilon} \text{Diagram}_{15}$$

$$\frac{1}{2 \times 3} \left( 2 \text{Diagram}_{16} + \text{Diagram}_{17} \right) = -\frac{\lambda^2}{4} \frac{3}{4\epsilon} \text{Diagram}_{15}$$

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$$\frac{1}{2 \times 3} \left( 2 \text{Diagram}_{13} + \text{Diagram}_{14} \right) = - \frac{\lambda^2}{4} \frac{3}{4\epsilon} \text{Diagram}_5$$

dilatation operator: above sum multiplied by  $-4\epsilon$

$$\text{Diagram}_{15} + \text{Diagram}_{16} \rightarrow - \lambda^2 \sum_{l=1}^{2L} \left( - \frac{1}{2} \text{Diagram}_1 - \frac{1}{2} \text{Diagram}_2 - \text{Diagram}_5 + \text{Diagram}_6 \right)$$

# Field theory calculation: 2-loop warm-up

$SU(2) \times SU(2)$  sector:

$$\text{Diagram} = \frac{\lambda^2}{4} \frac{1}{\epsilon} \frac{1}{4} \left( -2 \text{Diagram}_1 + 4 \text{Diagram}_2 \right)$$

The diagram on the left is a two-loop diagram with two vertical lines (one red, one green) and two external legs (one red, one green) that cross each other. Diagram 1 is two parallel vertical lines (one red, one green). Diagram 2 is a two-loop diagram with two vertical lines (one red, one green) and two external legs (one red, one green) that cross each other.

$$\frac{1}{2} \left( \text{Diagram}_3 + \text{Diagram}_4 \right) = + \frac{\lambda^2}{4} \frac{1}{4\epsilon} \text{Diagram}_5$$

$$\frac{1}{2 \times 3} \left( 2 \text{Diagram}_6 + \text{Diagram}_7 \right) = - \frac{\lambda^2}{4} \frac{3}{4\epsilon} \text{Diagram}_8$$

Diagram 3 and 4 are two-loop diagrams with two vertical lines (one red, one green) and two external legs (one red, one green) that cross each other, with a white cloud-like shape on the left. Diagram 5 is two parallel vertical lines (one red, one green). Diagram 6 and 7 are two-loop diagrams with two vertical lines (one red, one green) and two external legs (one red, one green) that cross each other, with a red and green dot respectively on the left. Diagram 8 is two parallel vertical lines (one red, one green).

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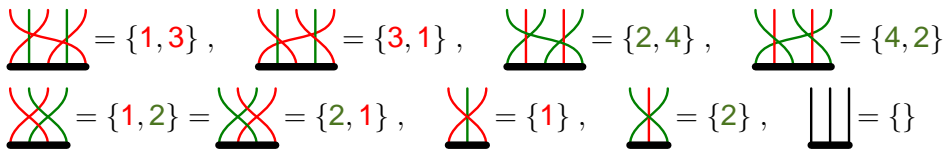
$$\text{Diagram}_9 + \text{Diagram}_{10} \rightarrow -\lambda^2 \sum_{l=1}^{2L} \left( - \text{Diagram}_{11} + \text{Diagram}_{12} \right)$$

Diagram 9 is two parallel vertical lines (one red, one green) with a red oval between them. Diagram 10 is two parallel vertical lines (one red, one green) with a green oval between them. Diagram 11 is two parallel vertical lines (one red, one green). Diagram 12 is a two-loop diagram with two vertical lines (one red, one green) and two external legs (one red, one green) that cross each other.

# Structure of the four-loop dilatation operator

permutation structures

$$\{a_1, a_2, \dots, a_m\} = \sum_{i=1}^L P_{2i+a_1} P_{2i+a_1+2} P_{2i+a_2} P_{2i+a_2+2} \dots P_{2i+a_m} P_{2i+a_m+2}$$





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ansatz:  $D_4 = D_4 + D_4 + D_4$

$$D_4 = c_5(\{1, 3\} + \{3, 1\}) + c_3\{1\} + c_0\{\}$$

$$D_4 = c_5(\{2, 4\} + \{4, 2\}) + c_3\{2\} + c_0\{\}$$

fixed by  $D_4 \mathcal{O}_{2L,0} = 0$

$$c_0 = -2c_5 - c_4 - c_3$$

$$D_4 = 2c_4\{1, 2\}$$

integrability:

$$c_4 = 0$$

we only have to calculate structures with permutations

# Vertex combinations at four loops

$$\begin{array}{cccc}
 V_{\psi^2 Y^2} = \text{X} & , & V_{\psi Y \psi Y} = \text{X} & , & V_{AYAY} = \text{X} & , & V_{A^2 Y^2} = \text{X} \\
 V_{\psi^2 A} = \text{---} & , & V_{Y^2 A} = \text{---} & , & V_{A^3} = \text{---}
 \end{array}$$

$$(V_{Y^6})^2,$$

$$V_{Y^6}(V_{\psi^2 Y^2})^2, V_{Y^6}(V_{A^2 Y^2})^2,$$

$$V_{Y^6} V_{\psi^2 Y^2} V_{\psi^2 A} V_{Y^2 A}, V_{Y^6} V_{AYAY} (V_{Y^2 A})^2, V_{Y^6} V_{A^2 Y^2} (V_{Y^2 A})^2, V_{Y^6} V_{A^2 Y^2} V_{Y^2 A} V_{A^3},$$

$$V_{Y^6} (V_{\psi^2 A})^2 (V_{Y^2 A})^2, V_{Y^6} (V_{Y^2 A})^4, V_{Y^6} (V_{Y^2 A})^3 V_{A^3}, V_{Y^6} (V_{Y^2 A})^2 (V_{A^3})^2,$$

$$(V_{\psi Y \psi Y})^2 (V_{\psi^2 Y^2})^2, (V_{\psi^2 Y^2})^4,$$

$$(V_{\psi^2 Y^2})^3 (V_{\psi^2 A})^2, (V_{\psi^2 Y^2})^3 V_{\psi^2 A} V_{Y^2 A}, (V_{\psi^2 Y^2})^3 (V_{Y^2 A})^2.$$

# Vertex combinations at four loops

$$\begin{aligned}
 V_{\psi^2 Y^2} &= \text{diagram 1}, & V_{\psi Y \psi Y} &= \text{diagram 2}, & V_{AYAY} &= \text{diagram 3}, & V_{A^2 Y^2} &= \text{diagram 4}, \\
 V_{\psi^2 A} &= \text{diagram 5}, & V_{Y^2 A} &= \text{diagram 6}, & V_{A^3} &= \text{diagram 7}
 \end{aligned}$$

range **five**, **four**, three

$$(V_{Y^6})^2,$$

$$V_{Y^6}(V_{\psi^2 Y^2})^2, V_{Y^6}(V_{A^2 Y^2})^2,$$

$$V_{Y^6} V_{\psi^2 Y^2} V_{\psi^2 A} V_{Y^2 A}, V_{Y^6} V_{AYAY} (V_{Y^2 A})^2, V_{Y^6} V_{A^2 Y^2} (V_{Y^2 A})^2, V_{Y^6} V_{A^2 Y^2} V_{Y^2 A} V_{A^3},$$

$$V_{Y^6} (V_{\psi^2 A})^2 (V_{Y^2 A})^2, V_{Y^6} (V_{Y^2 A})^4, V_{Y^6} (V_{Y^2 A})^3 V_{A^3}, V_{Y^6} (V_{Y^2 A})^2 (V_{A^3})^2,$$

$$(V_{\psi Y \psi Y})^2 (V_{\psi^2 Y^2})^2, (V_{\psi^2 Y^2})^4,$$

$$(V_{\psi^2 Y^2})^3 (V_{\psi^2 A})^2, (V_{\psi^2 Y^2})^3 V_{\psi^2 A} V_{Y^2 A}, (V_{\psi^2 Y^2})^3 (V_{Y^2 A})^2.$$

# Range five and four, complete $(V_{Y_6})^2$

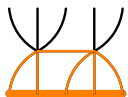
non-trace non-identity contributions, only  $\frac{1}{\epsilon}$  poles

$$\left. \begin{array}{l}
 \text{Diagram 1} + \text{refl.} \rightarrow (\{1, 3\} + \{3, 1\} - 2\{1\}), \\
 \text{Diagram 2} + \text{refl.} + \text{Diagram 3} \rightarrow 2(\{1, 2\} - \{1\})
 \end{array} \right\} \text{[Bak, Min, Rey]}$$
  

$$\text{Diagram 4} \rightarrow \frac{1}{2}\{1\}, \quad \text{Diagram 5} + \text{refl.} \rightarrow 2(-\{1\})$$

# Range five and four, complete $(V_{Y_6})^2$

non-trace non-identity contributions, only  $\frac{1}{\epsilon}$  poles



$$+ \text{refl.} \rightarrow (\{1, 3\} + \{3, 1\} - 2\{1\}),$$



$$+ \text{refl.} + \text{[diagram]} \rightarrow$$



$$2(\{1, 2\} - \{1\})$$

[Bak, Min, Rey]



$$\rightarrow$$

$$\frac{1}{2}\{1\},$$



$$+ \text{refl.} \rightarrow$$

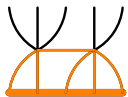
$$\frac{\lambda^4}{16} \frac{2}{\epsilon} 2(-\{1\})$$

integrals:



# Range five and four, complete $(V_{Y_6})^2$

non-trace non-identity contributions, only  $\frac{1}{\epsilon}$  poles



$$+ \text{refl.} \rightarrow (\{1, 3\} + \{3, 1\} - 2\{1\}),$$



$$+ \text{refl.} +$$



$$\rightarrow$$

$$2(\{1, 2\} - \{1\})$$

[Bak, Min, Rey]



$$\rightarrow$$

$$\frac{1}{2}\{1\},$$



$$+ \text{refl.} \rightarrow$$

$$\frac{\lambda^4}{16} \frac{2}{\epsilon} 2(-\{1\})$$

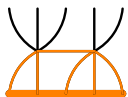
integrals:



$$= G(1, 1)^2$$

# Range five and four, complete $(V_{Y_6})^2$

non-trace non-identity contributions, only  $\frac{1}{\epsilon}$  poles



+ refl.  $\rightarrow$

$$(\{1, 3\} + \{3, 1\} - 2\{1\}),$$



+ refl. +



$\rightarrow$

$$2(\{1, 2\} - \{1\})$$

[Bak, Min, Rey]



$\rightarrow$

$$\frac{1}{2}\{1\},$$



+ refl.  $\rightarrow$

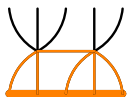
$$\frac{\lambda^4}{16} \frac{2}{\epsilon} 2(-\{1\})$$

integrals:

$$\frac{1}{2+\epsilon} \int_{\frac{1}{2+\epsilon}}^1 = G(1, 1)^2 G\left(\frac{1}{2} + \epsilon, 1\right)$$

# Range five and four, complete $(V_{Y_6})^2$

non-trace non-identity contributions, only  $\frac{1}{\epsilon}$  poles



+ refl.  $\rightarrow$

$$(\{1, 3\} + \{3, 1\} - 2\{1\}),$$



+ refl. +



$\rightarrow$

$$2(\{1, 2\} - \{1\})$$

[Bak, Min, Rey]



$\rightarrow$

$$\frac{1}{2}\{1\},$$



+ refl.  $\rightarrow$

$$\frac{\lambda^4}{16} \frac{2}{\epsilon} 2(-\{1\})$$

integrals:

$$\frac{1}{2+\epsilon} \nabla_{2\epsilon} = G(1, 1)^2 G(\frac{1}{2} + \epsilon, 1)$$



# Range five and four, complete ( $V_{Y_6}$ )<sup>2</sup>

non-trace non-identity contributions, only  $\frac{1}{\epsilon}$  poles

$$\left. \begin{array}{l}
 \text{Diagram 1} + \text{refl.} \rightarrow (\{1, 3\} + \{3, 1\} - 2\{1\}), \\
 \text{Diagram 2} + \text{refl.} + \text{Diagram 3} \rightarrow 2(\{1, 2\} - \{1\})
 \end{array} \right\} \text{[Bak, Min, Rey]}$$

$$\text{Diagram 4} \rightarrow \frac{1}{2}\{1\}, \quad \text{Diagram 5} + \text{refl.} \rightarrow \frac{\lambda^4}{16} \frac{2}{\epsilon} 2(-\{1\})$$

integrals:

$$\int_{\frac{1}{2}+3\epsilon} = G(1, 1)^2 G(\frac{1}{2} + \epsilon, 1) G(\frac{1}{2} + 3\epsilon, 1)$$

# Range five and four, complete ( $V_{Y_6}$ )<sup>2</sup>

non-trace non-identity contributions, only  $\frac{1}{\epsilon}$  poles

$$\left. \begin{aligned}
 & \text{Diagram 1} + \text{refl.} \rightarrow \frac{\lambda^4}{16} \frac{2}{\epsilon} (\{1, 3\} + \{3, 1\} - 2\{1\}), \\
 & \text{Diagram 2} + \text{refl.} + \text{Diagram 3} \rightarrow \frac{\lambda^4}{16} \left( \frac{2}{\epsilon} \right) 2(\{1, 2\} - \{1\}) \\
 & \text{Diagram 4} \rightarrow \frac{1}{2} \{1\}, \quad \text{Diagram 5} + \text{refl.} \rightarrow \frac{\lambda^4}{16} \frac{2}{\epsilon} 2(-\{1\})
 \end{aligned} \right\} \text{[Bak, Min, Rey]}$$

integrals:

$$\text{Diagram 6} = G(1, 1)^2 G\left(\frac{1}{2} + \epsilon, 1\right) G\left(\frac{1}{2} + 3\epsilon, 1\right) \rightarrow \frac{\lambda^4}{16} \left( -\frac{1}{2\epsilon^2} + \frac{2}{\epsilon} \right)$$

# Range five and four, complete ( $V_{Y_6}$ )<sup>2</sup>

non-trace non-identity contributions, only  $\frac{1}{\epsilon}$  poles

$$\left. \begin{aligned}
 & \text{Diagram 1} + \text{refl.} \rightarrow \frac{\lambda^4}{16} \frac{2}{\epsilon} (\{1, 3\} + \{3, 1\} - 2\{1\}), \\
 & \text{Diagram 2} + \text{refl.} + \text{Diagram 3} \rightarrow \frac{\lambda^4}{16} \left( \frac{2}{\epsilon} - \frac{2}{\epsilon} \right) 2(\{1, 2\} - \{1\})
 \end{aligned} \right\} \text{[Bak, Min, Rey]}$$

$$\text{Diagram 4} \rightarrow \frac{1}{2} \{1\}, \quad \text{Diagram 5} + \text{refl.} \rightarrow \frac{\lambda^4}{16} \frac{2}{\epsilon} 2(-\{1\})$$

integrals:

$$\begin{aligned}
 & \text{Diagram 6} = G(1, 1)^2 G\left(\frac{1}{2} + \epsilon, 1\right) G\left(\frac{1}{2} + 3\epsilon, 1\right) \rightarrow \frac{\lambda^4}{16} \left( -\frac{1}{2\epsilon^2} + \frac{2}{\epsilon} \right) \\
 & \text{Diagram 7} = G(1, 1) G\left(\frac{1}{2} + 3\epsilon, 1\right) \text{Diagram 8} \rightarrow \frac{\lambda^4}{16} \left( -\frac{2}{\epsilon} \right)
 \end{aligned}$$

# Range five and four, complete ( $V_{Y_6}$ )<sup>2</sup>

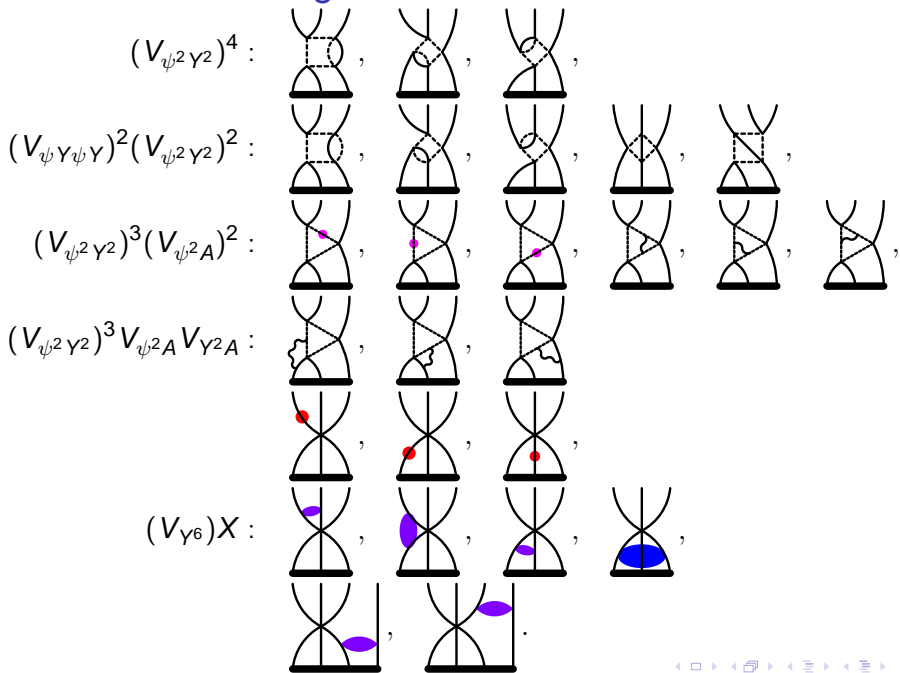
non-trace non-identity contributions, only  $\frac{1}{\epsilon}$  poles

$$\left. \begin{aligned}
 & \text{Diagram 1} + \text{refl.} \rightarrow \frac{\lambda^4}{16} \frac{2}{\epsilon} (\{1, 3\} + \{3, 1\} - 2\{1\}), \\
 & \text{Diagram 2} + \text{refl.} + \text{Diagram 3} \rightarrow \frac{\lambda^4}{16} \left( \frac{2}{\epsilon} - \frac{2}{\epsilon} \right) 2(\{1, 2\} - \{1\}) \\
 & \text{Diagram 4} \rightarrow \frac{\lambda^4}{16} \frac{1}{4\epsilon} \frac{1}{2} \{1\}, \quad \text{Diagram 5} + \text{refl.} \rightarrow \frac{\lambda^4}{16} \frac{2}{\epsilon} 2(-\{1\})
 \end{aligned} \right\} \text{[Bak, Min, Rey]}$$

integrals:

$$\begin{aligned}
 & \text{Diagram 1} = G(1, 1)^2 G\left(\frac{1}{2} + \epsilon, 1\right) G\left(\frac{1}{2} + 3\epsilon, 1\right) \rightarrow \frac{\lambda^4}{16} \left( -\frac{1}{2\epsilon^2} + \frac{2}{\epsilon} \right) \\
 & \text{Diagram 2} = G(1, 1) G\left(\frac{1}{2} + 3\epsilon, 1\right) \text{Diagram 3} \rightarrow \frac{\lambda^4}{16} \left( -\frac{2}{\epsilon} \right) \\
 & \text{Diagram 4} = G(1, 1)^3 G\left(\frac{1}{2} + \epsilon, 1 + 2\epsilon\right) \rightarrow \frac{\lambda^4}{16} \frac{3}{\epsilon} \zeta(2)
 \end{aligned}$$

# Range three interactions





# Flavour structures

$$\begin{array}{c} A \\ B \quad \diagup \quad \diagdown \\ \diagdown \quad \diagup \\ C \quad \diagdown \quad \diagup \\ \quad D \\ \quad \quad E \end{array} = (N_f - 6) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) - 8 \times$$

$$\begin{array}{c} B \quad A \quad H \\ \diagdown \quad \diagup \quad \diagdown \\ \diagdown \quad \diagup \quad \diagdown \\ C \quad \diagdown \quad \diagup \quad G \\ \quad D \quad \quad \quad E \quad \quad F \end{array} = (N_f - 8) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right)$$

$$+ 4 \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) + 4 \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) - 8 \times - 8 \times - 8 \times - 8 \times$$

$$+ 16 \times$$

$$\begin{array}{c} F \\ A \quad \diagup \quad \diagdown \\ \diagdown \quad \diagup \\ B \quad \diagdown \quad \diagup \\ \quad C \\ \quad \quad D \\ \quad \quad \quad E \end{array} = -2 \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) + 4 \times - 4 \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right)$$

$$\begin{array}{c} B \quad A \quad F \\ \diagdown \quad \diagup \quad \diagdown \\ \diagdown \quad \diagup \quad \diagdown \\ C \quad \diagdown \quad \diagup \quad E \\ \quad D \end{array} = -(2N_f - 10) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) + 8 \times - 8 \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right)$$

$$\begin{array}{c} B \quad A \quad F \\ \diagdown \quad \diagup \quad \diagdown \\ \diagdown \quad \diagup \quad \diagdown \\ C \quad \diagdown \quad \diagup \quad E \\ \quad D \end{array} = -(5N_f - 24) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) - 16 \times + 20 \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right) \left( \begin{array}{c} \diagdown \\ \diagup \end{array} \right)$$

# Effective Feynman rules

gauge boson propagators have numerator with  $\epsilon$ -tensor

$$\langle A_\alpha(p) A_\gamma(p) \rangle = -\langle \hat{A}_\alpha(p) \hat{A}_\gamma(p) \rangle = -\frac{2\pi}{k} \frac{1}{p^2} \epsilon_{\alpha\beta\gamma} p^\beta$$

$\Rightarrow$  effective Feynman rules

$$\text{wavy line } \mu \rightarrow 2i \begin{array}{c} \uparrow^\mu \\ | \\ \hline \end{array} = 2i \begin{array}{c} | \\ \hline \uparrow^\mu \end{array}$$

allow for manipulations directly on the graphs

$$\text{wavy line with loop} = \frac{1}{2} \left( \begin{array}{c} \uparrow \\ | \\ \hline \end{array} \begin{array}{c} \leftarrow \\ \circlearrowleft \\ \rightarrow \end{array} \begin{array}{c} | \\ \hline \uparrow \end{array} - \begin{array}{c} \uparrow \\ | \\ \hline \end{array} \begin{array}{c} \rightarrow \\ \circlearrowright \\ \leftarrow \end{array} \begin{array}{c} | \\ \hline \uparrow \end{array} \right) = 0,$$

$$\text{wavy line with triangle} = \begin{array}{c} \uparrow \\ | \\ \hline \end{array} \begin{array}{c} \rightarrow \\ \curvearrowright \\ \uparrow \end{array} - \begin{array}{c} \uparrow \\ | \\ \hline \end{array} \begin{array}{c} \rightarrow \\ \curvearrowleft \\ \uparrow \end{array} = 0,$$

$$\text{wavy line with triangle} = 2 \left( - \begin{array}{c} \uparrow \\ | \\ \hline \end{array} \begin{array}{c} \rightarrow \\ \curvearrowright \\ \uparrow \end{array} + \begin{array}{c} \uparrow \\ | \\ \hline \end{array} \begin{array}{c} \rightarrow \\ \curvearrowleft \\ \uparrow \end{array} \right) = 0.$$



# Effective Feynman rules

$$\text{Diagram 1} = \frac{1}{2} \text{Diagram 2},$$

$$\text{Diagram 3} = - \text{Diagram 4} + \text{Diagram 5},$$

$$\begin{aligned} \text{Diagram 6} = & \text{Diagram 7} - \text{Diagram 8} - \text{Diagram 9} + \text{Diagram 10} \\ & - \text{Diagram 11} + \text{Diagram 12} + \text{Diagram 13} - \text{Diagram 14}, \end{aligned}$$

$$\text{Diagram 15} = -4 \left( - \text{Diagram 16} + \text{Diagram 17} + \text{Diagram 18} - \text{Diagram 19} - \text{Diagram 20} + \text{Diagram 21} \right),$$

$$\text{Diagram 22} = 2 \left( \text{Diagram 23} - \text{Diagram 24} + \text{Diagram 25} + \text{Diagram 26} - \text{Diagram 27} \right),$$

$$\text{Diagram 28} = -4 \left( \text{Diagram 29} - \text{Diagram 30} \right).$$

## From ABJM to ABJ

gauge groups:  $U(N) \times \overline{U(N)} \rightarrow U(M) \times \overline{U(N)}$

't Hooft coupling constants:  $\lambda = \frac{M}{k}$  and  $\bar{\lambda} = \frac{N}{k}$

two-loops:  $\lambda^2 \rightarrow \lambda \bar{\lambda}$

[Bak, Gang, Rey]

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[Bak, Gang, Rey]

two loops with a common matter propagator:  $\rightarrow \lambda \bar{\lambda}$

trafo rule:

loop with a  $A$  or  $\hat{A}$  gauge boson propagator:  $\rightarrow \begin{cases} \lambda & \langle AA \rangle \\ \bar{\lambda} & \langle \hat{A}\hat{A} \rangle \end{cases}$

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finite one-loop fermion wave function renormalization (parity breaking):

$$\text{---}\bullet\text{---} = \text{---}\text{---} + \text{---}\text{---} = i(M - N) \text{---}\text{---}$$

$\Rightarrow$  log. divergent four-loop fermion triangle graphs ( $V_{\psi^2 \gamma^2}$  parity breaking):

$$\left. \begin{aligned} \text{---}\bullet\text{---} &= (M - N) M^2 N \frac{1}{4} \text{---}\text{---} \\ \text{---}\bullet\text{---} &= -(M - N) M N^2 \frac{1}{4} \text{---}\text{---} \end{aligned} \right\} \rightarrow \frac{1}{4} (M - N)^2 M N \frac{1}{16k} \left( -\frac{1}{4\epsilon^2} \right)$$

$\Rightarrow$  parity conserving interaction

# Symmetries

transformation of a Feynman graph by reflection and shifts *odd*  $\leftrightarrow$  *even*

$$c(M, N)\{a_1, \dots, a_m\} \rightarrow (-1)^{P_{A^2} + V_{AY^2} + V_{A\psi^2} + V_{Y^2\psi^2}} c(N, M)\{a_m, \dots, a_1\}$$

$$c(M, N)\{a_1, \dots, a_m\} \rightarrow (-1)^{P_{A^2} + V_{A^3} + V_{A\psi^2} + V_{Y^2\psi^2}} c(N, M)\{a_1 + 1, \dots, a_m + 1\}$$

$P_x, V_x$ : number of propagators, vertices of type  $x$

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four-loop graphs do not acquire signs

$$m \leq 1 : c(M, N) = c(N, M)$$

$$m \geq 2 : \begin{array}{ll} \text{reflection:} & c(M, N) = c(N, M) \\ \text{shift:} & c(M, N) = c(N, M) \end{array}$$

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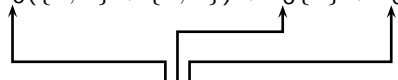
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symmetries of the dilatation operator:  $D_4 = D_4 + D_4$

$$D_4 = c_5(\{1, 3\} + \{3, 1\}) + c_3\{1\} + c_0\{ \}$$

$$D_4 = c_5(\{2, 4\} + \{4, 2\}) + c_3\{2\} + c_0\{ \}$$



$$c_i(M, N) = c_i(N, M)$$

$$\Rightarrow h(\lambda, \bar{\lambda}) = h(\bar{\lambda}, \lambda)$$

## What made us suffer

gauge boson propagators have numerator with  $\epsilon$ -tensor:

- ⇒ contracted momenta can be far apart in the graph of the integral:  
completion of squares of momenta hard to apply
- ⇒ diagrams with gauge fields decompose into many scalar diagrams



$$I_{4g1} = \text{Diagram} = 4 \left( \text{Diagram 1} - \text{Diagram 2} - I_{422211} + 2I_{422212} - I_{422213} \right),$$

$$I_{422211} = \text{Diagram} = \frac{1}{2} \left( \text{Diagram 1} + 2 \text{Diagram 2} - \text{Diagram 3} + 2 \text{Diagram 4} \right),$$

$$I_{422212} = \text{Diagram} = \frac{1}{2} \left( \text{Diagram 1} - \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} - \text{Diagram 5} \right),$$

$$I_{422213} = \text{Diagram} = \frac{1}{2} \left( \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} - \text{Diagram 5} \right).$$

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simple **cubic vertices** in  $D = 3$  are not infrared safe:

$$\text{pole part} \left( \text{diagram with horizontal red line} \right) \neq \text{pole part} \left( \text{diagram with curved red line} \right)$$

⇒ cubic vertices become quartic vertices due to **external momentum**  
integration by parts requires cubic vertices  
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integration by parts requires cubic vertices  
completion of squares hard to apply

integrals with odd number of loops give half-integer propagator weights

⇒ uniqueness and integration by parts methods hard to apply

## Status of the calculation

- ▶ all diagrams have been calculated
- ▶  $\sim 160$  two-, three- and four-loop integrals have been computed: using integration by parts, conformal inversion, sewing and cutting, Gegenbauer polynomial  $x$ -space techniques
- ▶ several relations between integrals have been checked
- ▶ consistency with known two-loop and maximal range four-loop results
- ▶ cancellation of  $\frac{1}{\epsilon^2}$  poles in  $\ln Z$  to be checked
- ▶ sign of the fermion triangle graph to be fixed by computing the two-loop vertex renormalization of  $V_{Y^6}$
- ▶ everything to be rechecked and written up
- ▶ proposal of simple rational function [Gromov, Vieira] almost certainly **not** correct for  $h(\lambda)^2 = \lambda^2 + \lambda^4 h_4$

$$h_4 = a + b\zeta(2) < 0, \quad a, b \in \mathbb{Z}$$

$\zeta(2)$  appears already in simple four-loop integrals

- ▶ no parity breaking effects  $h(\lambda, \bar{\lambda}) = h(\bar{\lambda}, \lambda)$