Strings on Semi-symmetric Superspaces

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0907.????

“Integrability in gauge and string theory”, Potsdam, 29.06.2009

AdS/CFT correspondence

D=4

Yang-Mills theory with N=4 supersymmetry ⟺ String theory on $AdS_5 x S^5$ background

Maldacena'97
Gubser,Klebanov,Polyakov'98
Witten'98

D=3

N=6 Supersymmetric Chern-Simons-matter theory ⟺ String theory on $AdS_4 x CP^3$ background

Aharony,Bergman,Jafferis,Maldacena'08
Aharony,Bergman,Jafferis'08

- Describe a line of conformal theories:
  - ✓ ‘t Hooft coupling: $0 < \lambda < \infty$
  - ✓ string tension (radius^2/α'): $T \propto \sqrt{\lambda}$
- Exact string backgrounds
- Integrable: spin chains / sigma-models

$$\text{Super}\left(AdS_5 \times S^5\right) = PSU(2,2|4)/SO(4,1) \times SO(5)$$ Metsaev,Tseytlin'98

$$\text{Super}\left(AdS_4 \times CP^3\right) = OSp(6|4)/U(3) \times SO(3,1)$$ Arutyunov,Frolov'08
Stefanski'08

Possess Z_4 symmetry (are semi-symmetric cosets)

Berkovits,Bershadsky,Hauer,Zhukov,Zwiebach'99

guarantees integrability

Goal: catalog all semi-symmetric sigma-models that have

- β – function = 0
- central charge = 26 /very interesting/

  < 26 /interesting/

- Formulate the conditions*
  $\beta = 0$ and $c < 26$
  in the algebraic terms
- Use Serganova's classification of semi-symmetric superspaces to find all candidate* sigma-models with vanishing β – function and correct supercharge

*) Remark: I will compute the β – function and the central charge only to the leading order in α'.

Symmetric spaces

- homogeneous:

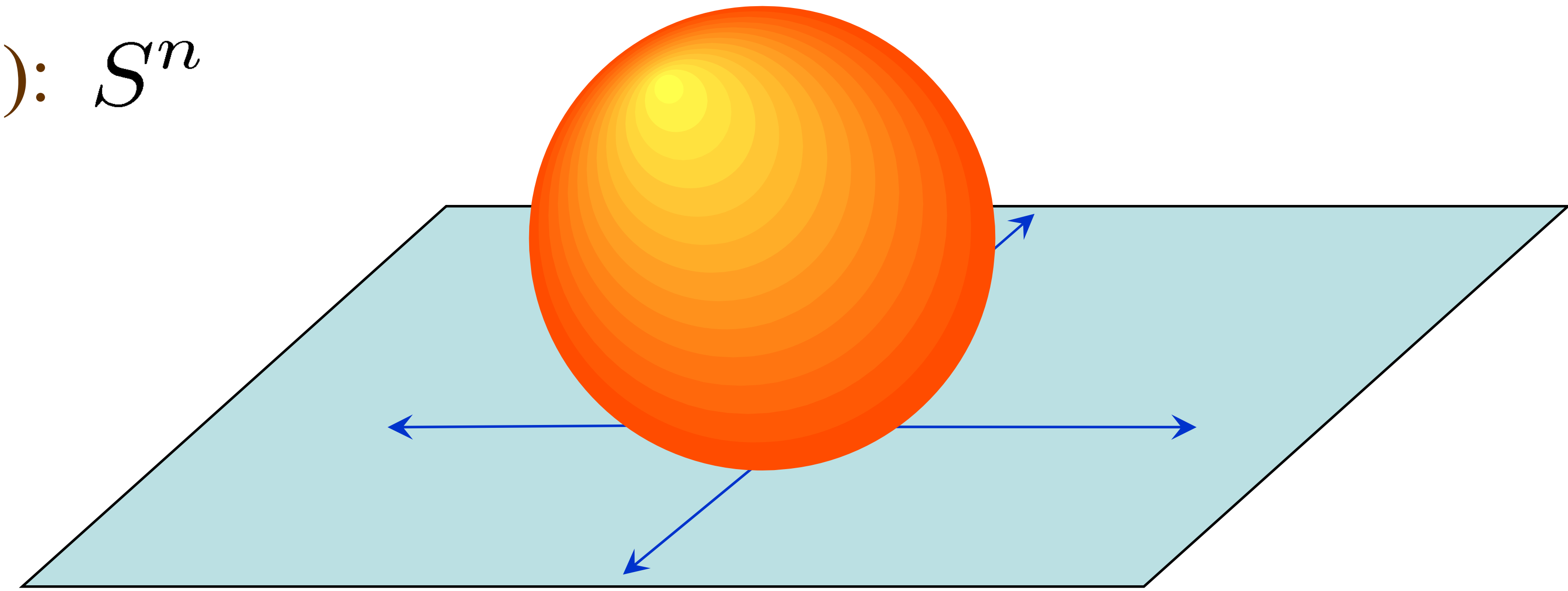
$$\mathcal{M} = G/H$$

- admit a Z_2 action (parity):

$$\Omega : \mathcal{M} \rightarrow \mathcal{M} \qquad\qquad \Omega^2 = \mathrm{id}$$

preserves the G-invariant metric

Ex (1): S^n



$$ds^2 = \frac{dz d\bar{z}}{(1+|z|^2)^2}$$

$$\Omega: \; z \to -z$$

Ex (2): AdS_n

G/H coset: $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{f}$

$[\mathfrak{h}, \mathfrak{h}] \subset \mathfrak{h}$ $\quad [T_a, T_b] = f_{abc} T_c$

$[\mathfrak{h}, \mathfrak{f}] \subset \mathfrak{f}$ $\quad [T_a, T_i] = f_{aij} T_j$

$\Rightarrow f_{abi} = 0$

$[\mathfrak{f}, \mathfrak{f}] \subset \mathfrak{h} \oplus \cancel{\mathfrak{f}}$ $\quad [T_i, T_j] = f_{ija} T_a + \cancel{f_{ijk} T_k}$

If coset G/H is symmetric

$\Updownarrow$

$f_{ijk} = 0$

Symmetric-space cosets

$$\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{f}$$

$$\begin{aligned} \Omega(\mathfrak{h}) &= \mathfrak{h} \\ \Omega(\mathfrak{f}) &= -\mathfrak{f} \end{aligned}$$

$$\begin{aligned} \Omega(T_a) &= T_a \\ \Omega(T_i) &= -T_i \end{aligned}$$

Coset representative: $g(X^i)$ e.g. $g = \mathrm{e}^{T_i X^i}$

Metric: $ds^2 = \mathrm{tr}\ (g^{-1}dg)_{\mathfrak{f}} (g^{-1}dg)_{\mathfrak{f}} = G_{ij}(X)dX^i dX^j$

Invariant under $X_i \to -X_i$

Sigma model

The sigma model on a symmetric coset:

$$\mathcal{L} = G_{ij}(X)\partial_\mu X^i \partial^\mu X^j$$

$$\mathcal{L} = \mathrm{tr}\,\left(g^{-1}\partial_\mu g\right)_f \left(g^{-1}\partial^\mu g\right)_f$$

- is (classically) integrable Eichenherr,Forger'81
- (for compact groups) is asymptotically free (β-function < 0)

Polyakov'75

Easily generalizes to Z_2 cosets of supergroups, which can be conformal (β-function = 0)!

Semi-symmetric cosets of supergroups

Serganova'83

- homogeneous superspace:

$$\mathcal{M} = G/H_0$$

- admits a Z_4 action:

$$\Omega : \mathcal{M} \to \mathcal{M} \qquad \Omega^4 = \mathrm{id}$$

Z_4 decomposition

$$\mathfrak{g} = \mathfrak{h}_0 \oplus \mathfrak{h}_1 \oplus \mathfrak{h}_2 \oplus \mathfrak{h}_3$$

$$\Omega(\mathfrak{h}_n) = \mathrm{e}^{i\pi n/2}\mathfrak{h}_n$$

$\mathfrak{h}_0, \mathfrak{h}_2$ - bosonic

$\mathfrak{h}_1, \mathfrak{h}_3$ - fermionic

$$[\mathfrak{h}_n, \mathfrak{h}_m] \subset \mathfrak{h}_{(n+m) \bmod 4}$$

Sigma model

g – coset representative:

$$g^{-1}\partial_\mu g = J_{\mu\,0} + J_{\mu\,1} + J_{\mu\,2} + J_{\mu\,3}.$$

Sigma-model Lagrangian:

$$\mathcal{L} = \frac{1}{2\kappa}\,\mathrm{Str}\left(\sqrt{h}h^{\mu\nu}J_{\mu\,2}J_{\nu\,2} + i\epsilon^{\mu\nu}J_{\mu\,1}J_{\nu\,3}\right)$$

Metsaev,Tseytlin'98
Roiban,Siegel'00

Automatically integrable! follows from Z_4 symmetry

Bena,Polchinski,Roiban'03

Background field method

Polyakov'75;04
Adam,Dekel,Mazzucato,Oz'07

Coset representative:

$$g = \bar{g}\,\mathrm{e}^{X} \qquad X \in \mathfrak{h}_1 \oplus \mathfrak{h}_2 \oplus \mathfrak{h}_3$$

Background gauge field:

$$A_\mu = \left(\bar{g}^{-1}\partial_\mu \bar{g}\right)_0 \qquad D_\mu = \partial_\mu + [A_\mu, \cdot]$$

Background current:

$$K_\mu = \left(\bar{g}^{-1}\partial_\mu \bar{g}\right)_2$$

Assumed to satisfy classical equations of motion

Second-order Lagrangian for fluctutations:

$$\mathcal{L}_2 = \frac{1}{2}\,\mathrm{Str}\left(\bar{D}X_2 D X_2 - [\bar{K}, X_2][K, X_2] + X_1 D[\bar{K}, X_1] + X_3 \bar{D}[K, X_3] - 2[K, X_3][\bar{K}, X_1]\right)$$

N_b massive bosons: $N_b = \dim \mathfrak{h}_2 - 2$

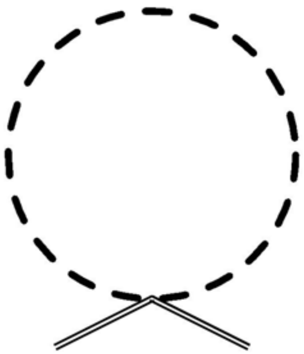
N_f massive fermions: $N_f = \frac{1}{2}\left(\dim \mathfrak{h}_1 - N_\kappa + \dim \mathfrak{h}_3 - N_{\tilde{\kappa}}\right)$

κ – symmetry: the Lagrangian does not depend on

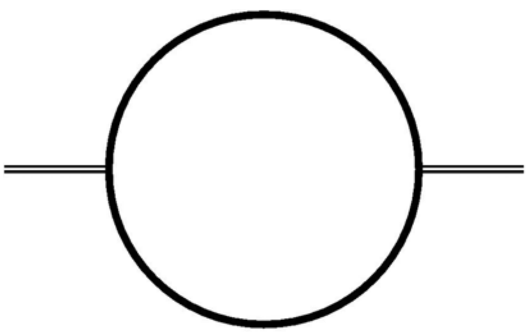
X_1 such that $[\bar{K}, X_1] = 0$ and on X_3 such that $[K, X_3] = 0$

$$N_\kappa = \dim \ker \mathrm{ad}\ \bar{K}\big|_{\mathfrak{h}_1} \qquad N_{\tilde{\kappa}} = \dim \ker \mathrm{ad}\ K\big|_{\mathfrak{h}_3}$$

β - function



$$\frac{1}{8\pi}\,\mathrm{tr}_2\left(\mathrm{ad}\,K\,\mathrm{ad}\,\bar{K}+\mathrm{ad}\,\bar{K}\,\mathrm{ad}\,K\right)\,\ln\Lambda$$



$$-\frac{1}{8\pi}\left(\mathrm{tr}_1\,\mathrm{ad}\,K\,\mathrm{ad}\,\bar{K}+\,\mathrm{tr}_3\,\mathrm{ad}\,\bar{K}\,\mathrm{ad}\,K\right)\ln\Lambda$$

Bare Lagrangian:

$$\bar{\mathcal{L}} = \frac{1}{2} G_{ij} K^i_\mu K^j_\mu \qquad G_{ij} = \frac{1}{\kappa} \mathrm{Str}\, T_i T_j$$

β – function:

$$\frac{dG_{ij}}{d \ln \Lambda} = \frac{1}{4\pi} (-1)^{|A|} f^A_{iB} f^B_{jA}$$

Killing form

β – function = 0

$\Updownarrow$

Killing form vanishes

Central charge

Each boson contributes 1 to c_L and 1 to c_R

Each fermion from $\mathfrak{h}_1$ contributes 2 to c_L

Each fermion from $\mathfrak{h}_3$ contributes 2 to c_R

Carlip'87
Kallosh,Morozov'88
Wiegmann'89

Possible to show that $c_L = c_R$

$$c = \dim \mathfrak{g} - \dim \mathfrak{h}_0 - N_\kappa - N_{\tilde{\kappa}}$$

$N_\kappa = \dim \ker \mathrm{ad}\, \bar{K}\big|_{\mathfrak{h}_1}$ $\quad$ $N_{\tilde{\kappa}} = \dim \ker \mathrm{ad}\, K\big|_{\mathfrak{h}_3}$

Example (1)

$$PSU(2,2|4)/SO(4,1)\times SO(5)$$

Killing form = 0

$c_b = 30 - 20 = 10$

$c_f = 32 - \underset{N_\kappa}{8} - \underset{N_{\tilde{\kappa}}}{8} = 16$

$c = 10 + 16 = 26$

Example (2)

$$OSp(6|4)/U(3) \times SO(3,1)$$

Killing form = 0

$$c_b = 25 - 15 = 10$$

$$c_f = 24 - \underbrace{4}_{N_\kappa} - \underbrace{4}_{N_{\tilde{\kappa}}} = 16$$

$$c = 10 + 16 = 26$$

Basic Lie superalgebras with vanishing Killing form

Unitary series: PSU(n|n)

Orthosymplectic series: OSp(2n+2|2n)

deformation of OSp(4|2)

Exceptional: D(2,1;α)

Semi-symmetric cosets of PSU(n|n)

Serganova'83

H_0

- type U1: $U(p) \times SU(n-p) \times U(q) \times SU(n-q)$
- type U2: $SO(n) \times SO(n)$
- type U3: $SU(n)$
- type U4: $Sp(n) \times Sp(n)$

Subcritical type U sigma-models

U3:	$PSU(1,1\|1,1)/SU(1,1)$	c = 7	AdS_3
U1/2:	$PSU(1,1\|2)/U(1)\times U(1)$	c = 8	$AdS_2\times S^2$
U3:	$PSU(3\|3)/SU(3)$	c = 20	$SU(3)$
U1:	$PSU(3\|3)/U(2)\times U(2)$	c = 22	$CP^2\times CP^2$
U4:	$PSU(2,4\|4)/SO(4,1)\times SO(5)$	c = 26	$AdS_5\times S^5$

Semi-symmetric cosets of OSp(2n+2|2n)

Serganova'83

H_0

- type O1: $SO(p) \times SO(2n+2-p) \times U(n)$

- type O2: $SU(n+1) \times Sp(2q) \times Sp(2n-2q)$

Subcritical type O sigma-models

O1: $OSp(4|2)/SO(3)\times U(1)$ c = 11 $AdS_2\times S^3$

O1: $OSp(4|2)/U(1)\times U(1)\times U(1)$ c = 14 $AdS_2\times S^2\times S^2$

O2: $OSp(6|4)/U(3)\times SO(3,1)$ c = 26 $AdS_4\times CP^3$

Tensor product models

Permutation operator:

$$P: \ \mathfrak{g} \otimes \mathfrak{g} \to \mathfrak{g} \otimes \mathfrak{g}$$

$$P \circ (X, Y) = (Y, X)$$

Z_4 symmetry:

$$\Omega = \left(\mathrm{id} \otimes (-1)^F\right) \circ P$$

$$\Omega^2 = (-1)^F \quad \Longrightarrow \quad \Omega^4 = \mathrm{id}$$

Invariant subgroup:

$$\mathfrak{h}_0 = \{(X, X) \, | X \in \mathfrak{g}_{\mathrm{bos}} \}$$

Subcritical tensor product sigma-models

UT: $PSU(1,1|2) \times PSU(1,1|2)/SU(1,1) \times SU(2)$

c = 14 $AdS_3 \times S^3$

OT: $OSp(4|2) \times OSp(4|2)/SO(4) \times Sp(2)$

c = 25 $AdS_3 \times S^3 \times S^3$

AdS/CFT

Critical models

AdS$_5$/CFT$_4$:

$$AdS_5 \times S^5$$

nothing else

AdS$_4$/CFT$_3$:

$$AdS_4 \times CP^3$$

nothing else

AdS_3/CFT_2:

1) $AdS_3 \times S^3$

central charge deficit: c = 14

$$AdS_3 \times S^3 \times T^4$$

Dual to symmetric orbifold CFT

Berkovits,Vafa,Witten'99
Berkovits'00

AdS_3/CFT_2:

2) $AdS_3 \times S^3 \times S^3$

c = 25

$AdS_3 \times S^3 \times S^3 \times S^1$

- no κ – symmetries
- Global OSp(4|2) x OSp(4|2) invariance
- Dual to a (4,4) CFT

Gukov,Martinec,Moore,Strominger'04

AdS_2/CFT_1:

1) $AdS_2 \times S^2 \times (?)$

Zhou'99
Berkovits,Bershadsky,Hauer,Zhukov,Zwiebach'99

2) $AdS_2 \times S^3 \times (?)$

3) $AdS_2 \times S^2 \times S^2 \times (?)$

Dual to superconformal D=1 matrix models?

Remarks

- There is a finite number of semisymmetric sigma-models with $c \leq 26$
- The number may be even smaller, because the β – function and the central charge were computed only at one loop.
- The only critical models and the only models dual to CFT_d with $d > 2$ are $AdS_5 x S^5$ and $AdS_4 x CP^3$.
- κ – symmetry is not generic; it arises only for some low-rank cosets.