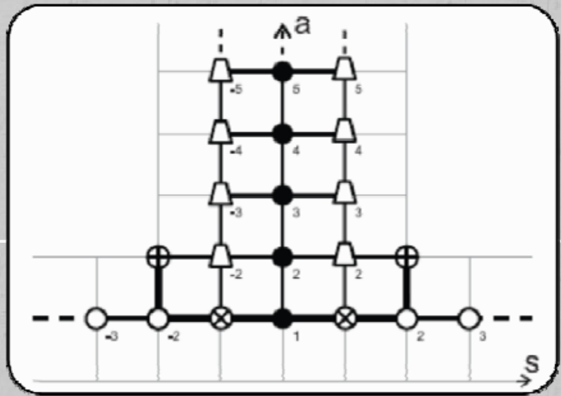
Nikolay Gromov

Based on works with V.Kazakov, P.Vieira & A.Kozak

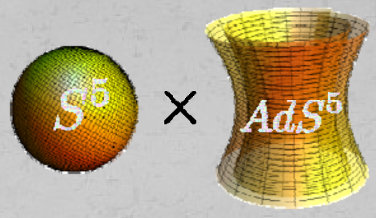


Konishi: from 4 loops to infinity

Integrability in Gauge and String Theory

AdS/CFT correspondence

AdS/CFT duality:



$$S = \frac{T}{2}\int \partial_\mu \vec{u} \cdot \partial^\mu \vec{u} \;\, d\sigma d\tau$$

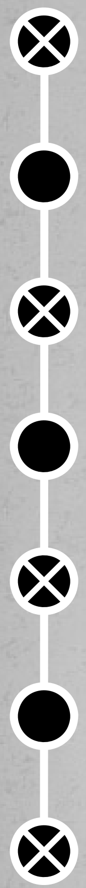
String tension $T = \frac{\sqrt{\lambda}}{4\pi} \equiv g$

`t Hooft coupling $\lambda = g_{YM}^2 N$

Anomalous dimensions = spectrum of 2D integrable field theories

Symmetry: $psu(2,2|4)$

Bethe equations



$$1 = \frac{Q_{2L}^{+} B^{(-)}}{Q_{2L}^{-} B^{(+)}}$$

$$-1 = \frac{Q_{2L}^{--} Q_{1L}^{+} Q_{3L}^{+}}{Q_{2L}^{++} Q_{1L}^{-} Q_{3L}^{-}}$$

$$1 = \frac{Q_{2L}^{+} R^{(-)}}{Q_{2L}^{-} R^{(+)}}$$

$$-1 = e^{-R\epsilon_1^*} \left(\frac{Q_4^{--}}{Q_4^{++}} \frac{B_{1L}^{+} R_{3L}^{+}}{B_{1L}^{-} R_{3L}^{-}} \frac{B_{1R}^{+} R_{3R}^{+}}{B_{1R}^{-} R_{3R}^{-}} \right) \left(\frac{B^{+(+)}}{B^{-(-)}} \right)^2 S^2$$

. . .

Beisert, Staudacher;
Beisert, Hernandez, Lopez;
Beisert,Eden,Staudacher
Arutyunov, Frolov

$$Q_l(u) = \prod_{j=1}^{J_l} (u - u_{l,j})$$

$$R_l^{(\pm)}(u) \equiv \prod_{j=1}^{K_l} \frac{x(u) - x_{l,j}^{\mp}}{\sqrt{x_{l,j}^{\mp}}}$$

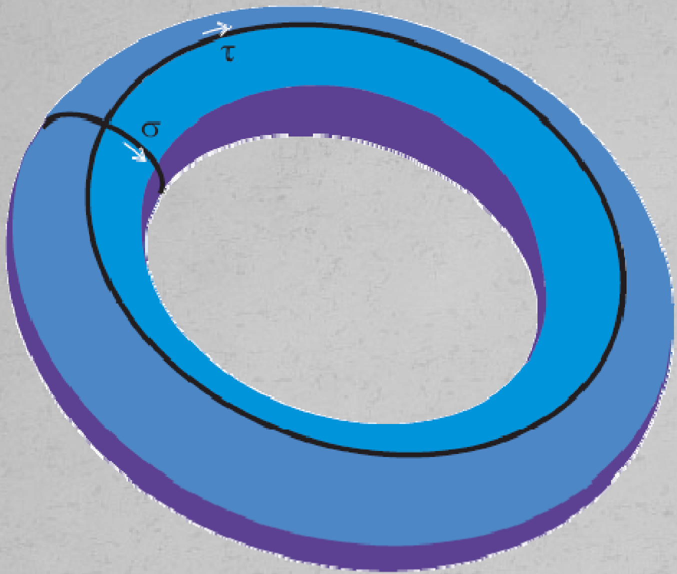
$$B_l^{(\pm)}(u) \equiv \prod_{j=1}^{K_l} \frac{\frac{1}{x(u)} - x_{l,j}^{\mp}}{\sqrt{x_{l,j}^{\mp}}};$$

$$x + \frac{1}{x} = \frac{u}{g} \;, \quad x^{\pm} + \frac{1}{x^{\pm}} = \frac{u \pm i/2}{g}$$

$$E = \sum_k \epsilon_k = \sum_k 2gi \left(\frac{1}{x_{4,k}^{+}} - \frac{1}{x_{4,k}^{-}} \right)$$

Vacuum

…,Matsubara, Zamolodchikov,…



$$Z(\tau,\sigma) = Z(\sigma,\tau)$$

$$\sum e^{-E_n(L)R} \qquad \sum e^{-E_n(R)L}$$

$$e^{-E_0(L)R}$$

I.e. from the asymptotical spectrum (infinite R) we can compute the Ground state energy for ANY finite volume!

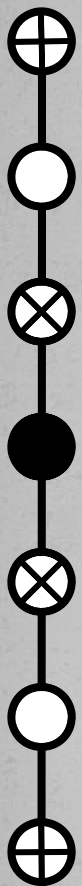
$$E_0(L) = -\lim_{R\to\infty} \frac{\log \sum e^{-E_n(R)L}}{R}$$

In our case the wick rotated theory is different – ”mirror” theory

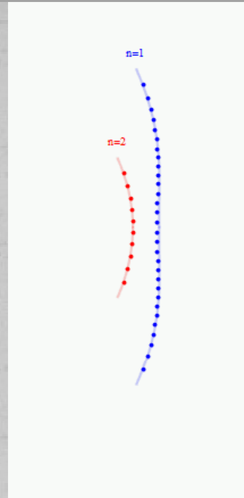
Ambjorn, Janik, Kristjansen; Arutynov, Frolov

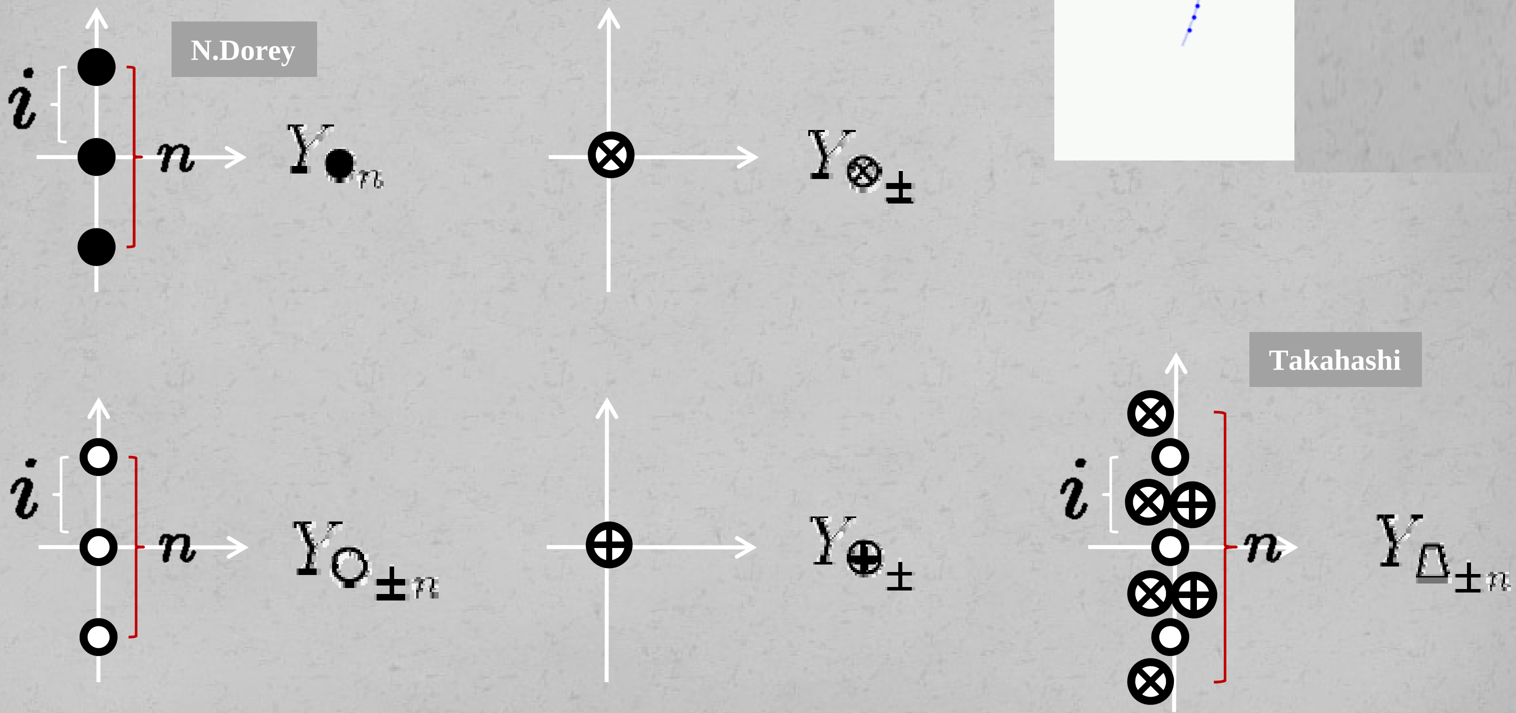
AdS/CFT bound states

Till Bargheer, Niklas Beisert, N. G.



$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k=1 (k\neq j)}^{J} \frac{u_j - u_k + i}{u_j - u_k - i}$$



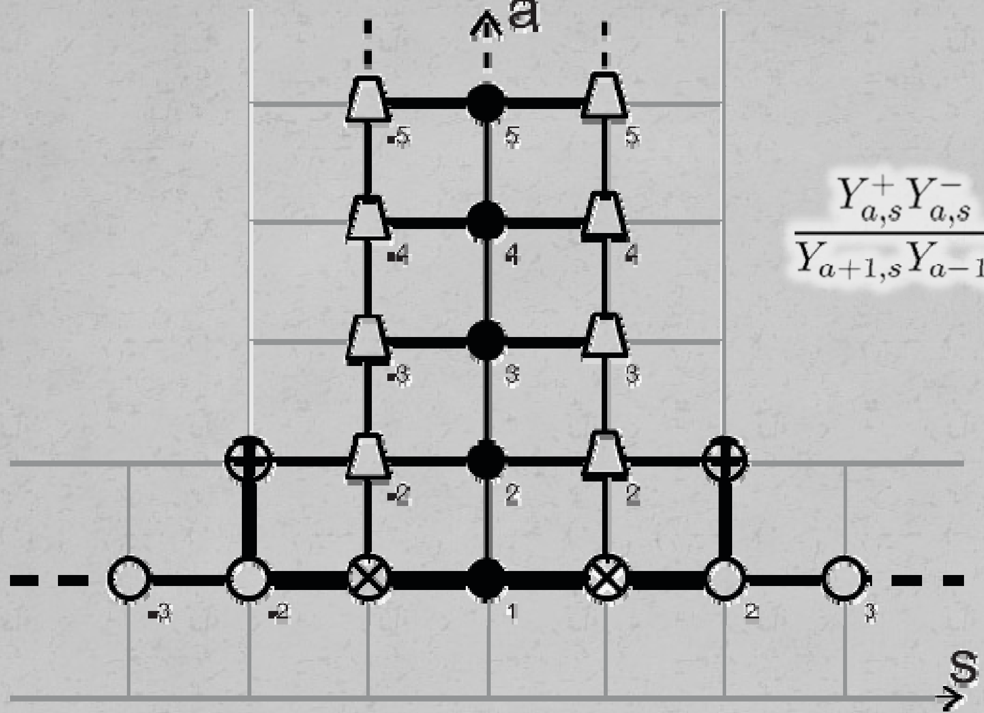


Adopted for mirror theory

Arutyunov, Frolov

AdS/CFT Y-system

N.G., Kazakov, Vieira



$$\frac{Y_{a,s}^{+}Y_{a,s}^{-}}{Y_{a+1,s}Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

$$E = \sum_{a=1}^{\infty} \int_{-\infty}^{\infty} \frac{du}{2\pi i} \frac{\partial \epsilon_a}{\partial u} \log\left(1 + Y_{a,0}(u)\right)$$

…Bazhanov, Lukyanov, Zamolodchikov, P.Dorey, Totteo…

…Destri de Vega, Bytsko ,Teschner….

Above ground state

Bazhanov, Lukyanov, Zamolodchikov, P.Dorey, Totteo

Inspiration #1: Analytical continuation

$$E = \sum_{a=1}^{\infty} \int_{-\infty}^{\infty} \frac{du}{2\pi i} \frac{\partial \epsilon_a}{\partial u} \log \left(1 + Y_{a,0}(u)\right)$$

$$\downarrow$$

$$E = \sum_{a=1}^{\infty} \int_{-\infty}^{\infty} \frac{du}{2\pi i} \frac{\partial \epsilon_a}{\partial u} \log \left(1 + Y_{a,0}(u)\right) + \sum_{a} \epsilon_j(u_{4,j})$$

$$\boldsymbol{Y_{1,0}(u_{4,j}) = -1}$$

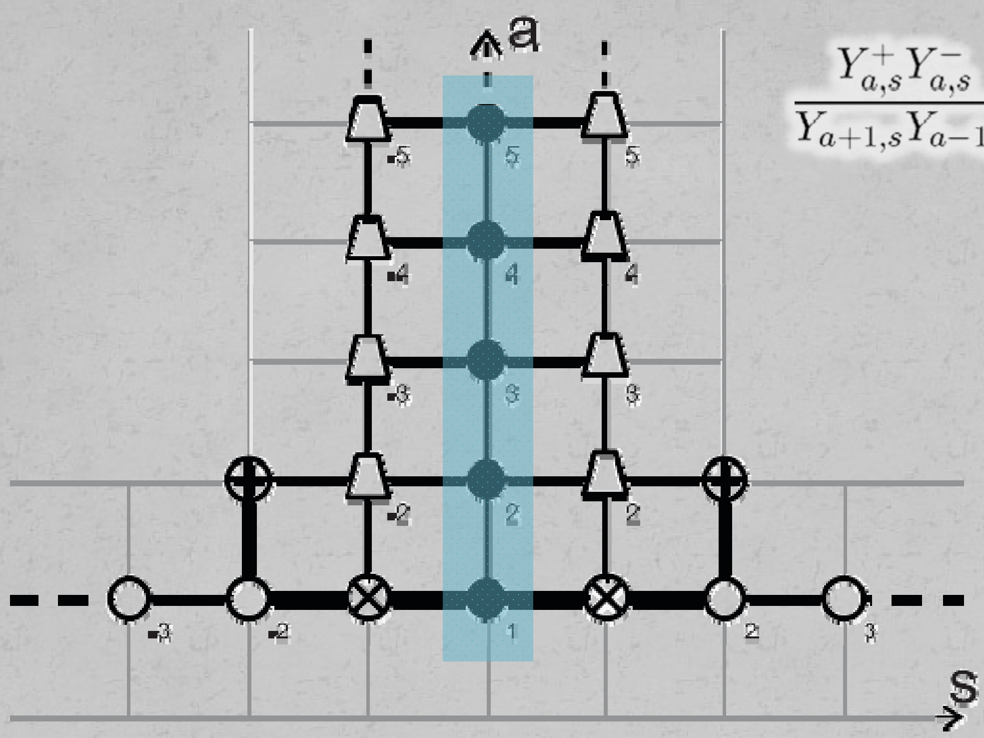
$$\frac{Y^+_{a,s} Y^-_{a,s}}{Y_{a+1,s} Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})} \rightarrow \frac{Y^+_{a,s} Y^-_{a,s}}{Y_{a+1,s} Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

Inspiration #2: Integrable lattice models

Destri de Vega, Teschner

Large L limit or weak coupling

N.G., Kazakov, Vieira



$$\frac{Y_{a,s}^{+}Y_{a,s}^{-}}{Y_{a+1,s}Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

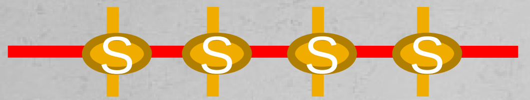
Use Hirota equation:

$$T_{a,s}^{+}T_{a,s}^{-} = T_{a+1,s}T_{a-1,s} + T_{a,s+1}T_{a,s-1}\,,$$

$$\text{where } Y_{a,s} = \frac{T_{a,s+1}T_{a,s-1}}{T_{a+1,s}T_{a-1,s}}\,.$$

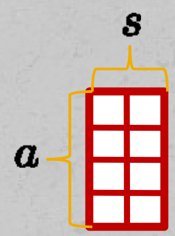
Hirota – large L

$$\hat{T}_{rep}(u|\theta_1,\theta_2,\ldots) = \mathrm{Tr}_{rep}(S(u,\theta_1)S(u,\theta_2)\ldots)$$



The eigevalues solve Hirota!

for rep =



Explicitly

Instead of computing the transfer matrices explicitly one can use:

$$\mathcal{W} = \left[1 - \frac{Q_1^- B^{+(+)} R^{-(+)}}{Q_1^+ B^{+(-)} R^{-(-)}} D\right] \left[1 - \frac{Q_1^- Q_2^{++} R^{-(+)}}{Q_1^+ Q_2 R^{-(-)}} D\right]^{-1} \times$$

$$\times \left[1 - \frac{Q_2^{--} Q_3^+ R^{-(+)}}{Q_2 Q_3^- R^{-(-)}} D\right]^{-1} \left[1 - \frac{Q_3^+}{Q_3^-} D\right] \;,\; D = e^{-i\partial_u}$$

$$\mathcal{W} = \sum_{s=0}^{\infty} T_{1,s}^{[1-s]} D^s$$

$$\mathcal{W}^{-1} = \sum_{a=0}^{\infty} (-1)^a T_{a,1}^{[1-a]} D^a$$

Beisert

By a short code we can generate any Tas

Bazhanov, Reshitikhin

```
T[aa_,ss_]:=Block[{TS,T1,T2,T3,T4,τ,j,uu},
{T1[u_],T2[u_],T3[u_],T4[u_]}={-1,Rp[u-I/2]/Rm[u-I/2],Rp[u-I/2]/Rm[u-I/2],-(Rp[u-I/2] Bp[u+I/2])/(Bm[u+I/2] Rm[u-I/2])};
T2[x_,a_,b_]:= Product[T2[x+I jj],{jj,a,b}];
T3[x_,a_,b_]:= Product[T3[x+I jj],{jj,a,b}];
τ[u_,j_]:=Sum[T2[u,-(j-1)/2,(j-2k-1)/2]T3[u,(j-2k+1)/2,(j-1)/2],{k,0,j}];
τ[u_,j_?(#<0&)]:=0;
TS[u_,s_]:=τ[u,s]+T1[u-I/2 (s-1)]τ[u+I/2,s-1]+τ[u-I/2,s-1]T4[u+I/2 (s-1)]+T1[u-I/2 (s-1)]τ[u,s-2]T4[u+I/2 (s-1)]/;s>=0;
TS[u_,s_]=0;
Table[TS[u+(aa+1-i-j)/2 I,ss+i-j],{i,aa},{j,aa}]//Det
]
```

Checked by explicit computation for some T's

Arutyunov, de Leeuw, Suzuki, Torrielli

Konishi operator

Bianchi, Kovacs,Rossi, Stanev; Eden, Jarczak, Sokatchev,Stanev…

The simplest operator

$$\mathcal{O} = tr(ZZWW) - tr(ZWZW) \qquad g \to 0$$

$$Y_{a,0} = g^8 \left(3\, 2^7 \frac{3a^3 + 12au^2 - 4a}{(a^2+4u^2)^2} \right)^2 \frac{1}{y_a(u) y_{-a}(u)}$$

$$y_a(u) = 9a^4 - 36a^3 + 72u^2a^2 + 60a^2 - 144u^2a - 48a + 144u^4 + 48u^2 + 16$$

$$(324 + 864\zeta_3 - 1440\zeta_5) g^8$$

In agreement with perturbation theory!!

4-loops!

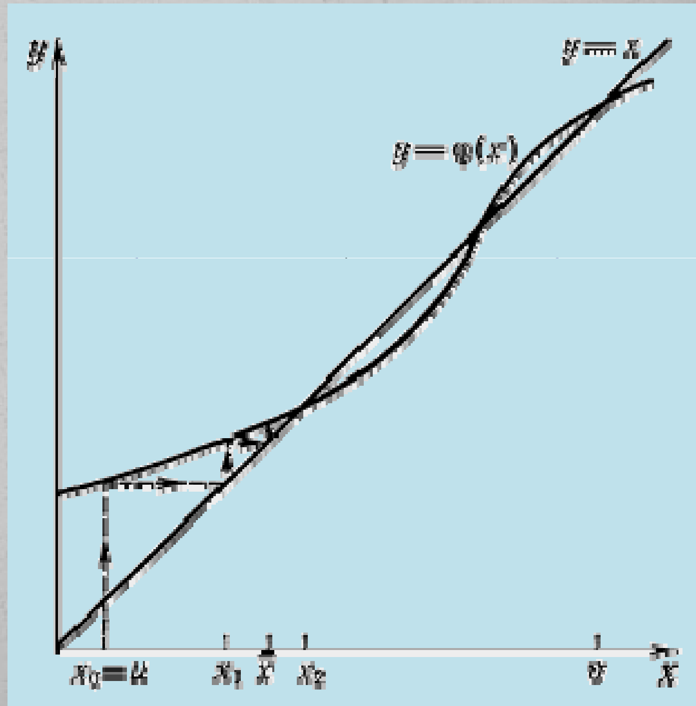
Kotikov, Lipatov, Rej, Staudacher and Velizhanin
Sieg, Torrielli; Janik, Bojnok,;
N.G., Kazakov, Vieira

Integral form of Y-system for exitations

$$\log Y_{\otimes} = K_{m-1} * \log(1 + 1/Y_{\circ_m})/(1 + Y_{\triangle_m})$$
$$+ \mathcal{R}^{(0m)} * \log(1 + Y_{\bullet_m}) \, .$$
$$\log Y_{\triangle_m} = \mathcal{M}_{nm} * \log(1 + Y_{\bullet_m}) - K_{n-1} \circledast \log(1 + Y_{\otimes})$$
$$- K_{n-1,m-1} * \log(1 + Y_{\triangle_m})$$
$$\log Y_{\circ_m} = K_{n-1,m-1} * \log(1 + 1/Y_{\circ_m}) + K_{n-1} \circledast \log(1 + Y_{\otimes})$$
$$\log Y_{\bullet_m} = \mathcal{T}_{nm} * \log(1 + Y_{\bullet_m}) + 2\mathcal{R}^{(n0)} \circledast \log(1 + Y_{\otimes})$$
$$+ \mathcal{N}_{nm} * \log(1 + Y_{\triangle_m}) \qquad (5)$$

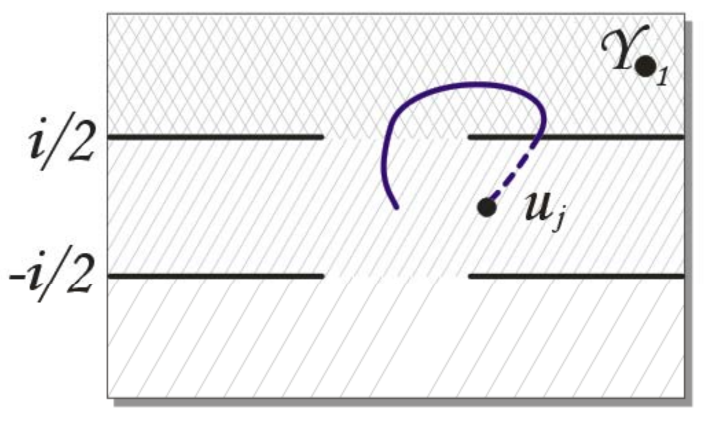
Bombardelli, Fioravanti, Tateo
N.G., Kazakov, Vieira
Arutynov, Frolov

$$Y_{a,s} = \frac{T_{a,s+1} T_{a,s-1}}{T_{a+1,s} T_{a-1,s}}$$



Exact BAE

- From mirror to physical



$$\log Y_{\bullet_n} = \mathcal{T}_{nm} * \log(1+Y_{\bullet_m}) + 2\mathcal{R}^{(n0)} \circledast \log(1+Y_{\otimes}) + \mathcal{N}_{nm} * \log(1+Y_{\square_m}) + i\Phi_n \, .$$

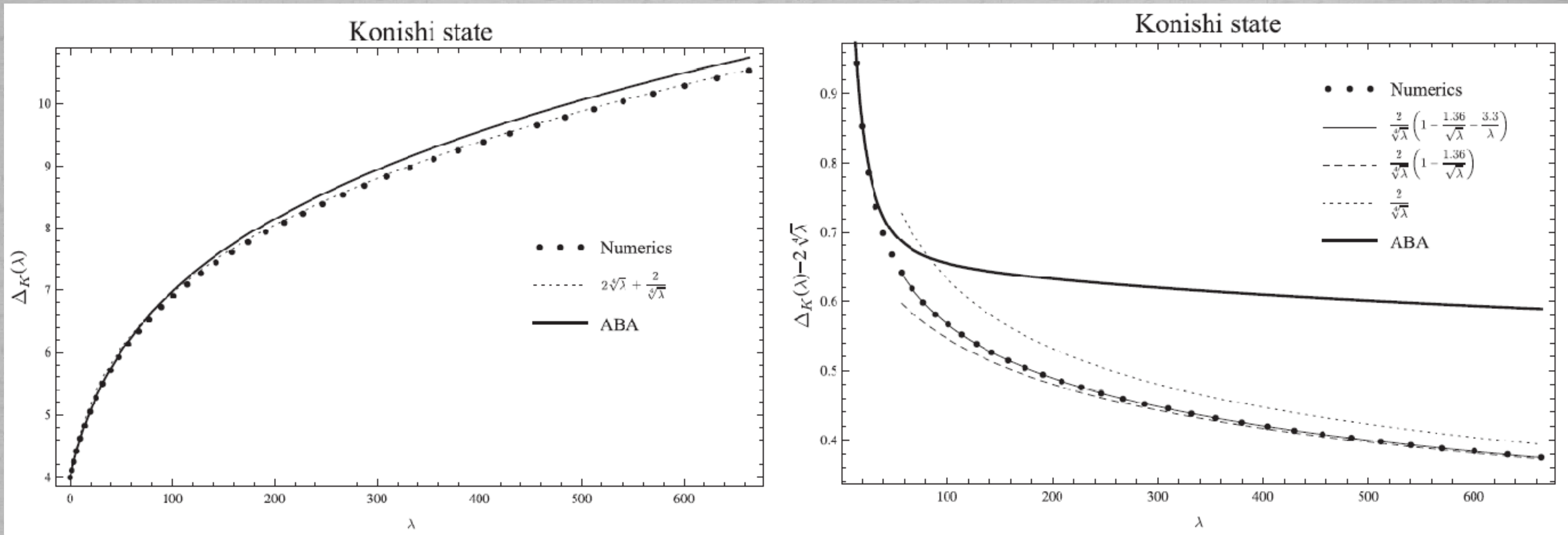
$$Y^{\mathrm{ph}}_{\bullet_1}(u_j) = -1$$

$$\log Y^{\mathrm{ph}}_{\bullet_1}(u_1) = \tilde{\mathcal{T}}_{1m} * \log(1+Y_{\bullet_m}) + \log Z_{\square_2}(u_1) + i\Phi_{\mathrm{ph}}(u_1) + 2(\mathcal{R}^{(10)}_{\mathrm{ph,mir}} \circledast K_{m-1} + K^{-}_{m-1}) *_{p.v.} \log(Z_{\square_m}) + 2\mathcal{R}^{(10)}_{\mathrm{ph,mir}} \circledast \log(1+Y_{\otimes}) - 2\log\frac{u_1 - i/2}{i} - 2\sum_{j=1}^{2} \log \frac{\frac{1}{x_1^+} - x_j^+}{\frac{1}{x_1^-} - x_j^+} \quad (6)$$

$0.000183047 + 0.875288\, i$

Precision is excellent!!!

Konishi – from weak to strong



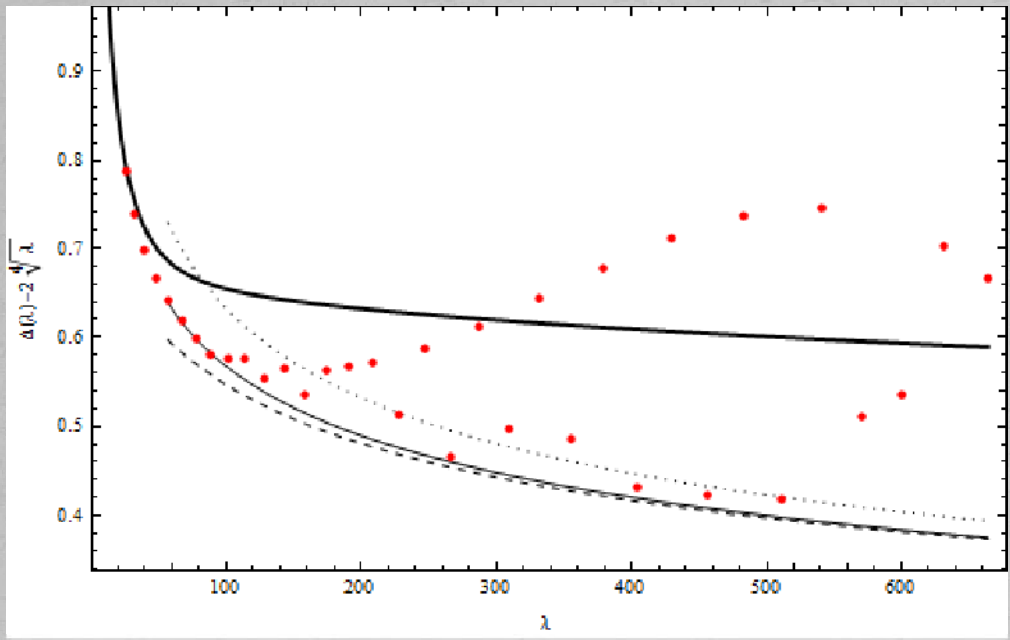
Fits:

$$2\sqrt[4]{\lambda}+0.0088+\frac{2.019}{\sqrt[4]{\lambda}}-\frac{0.8}{\sqrt{\lambda}}$$

$$2\sqrt[4]{\lambda}+0+\frac{1.9904}{\sqrt[4]{\lambda}}+\frac{0.284}{\sqrt{\lambda}}+\dots$$

$$2\sqrt[4]{\lambda}+0+\frac{2}{\sqrt[4]{\lambda}}+0-\frac{2.58}{\lambda^{3/4}}$$

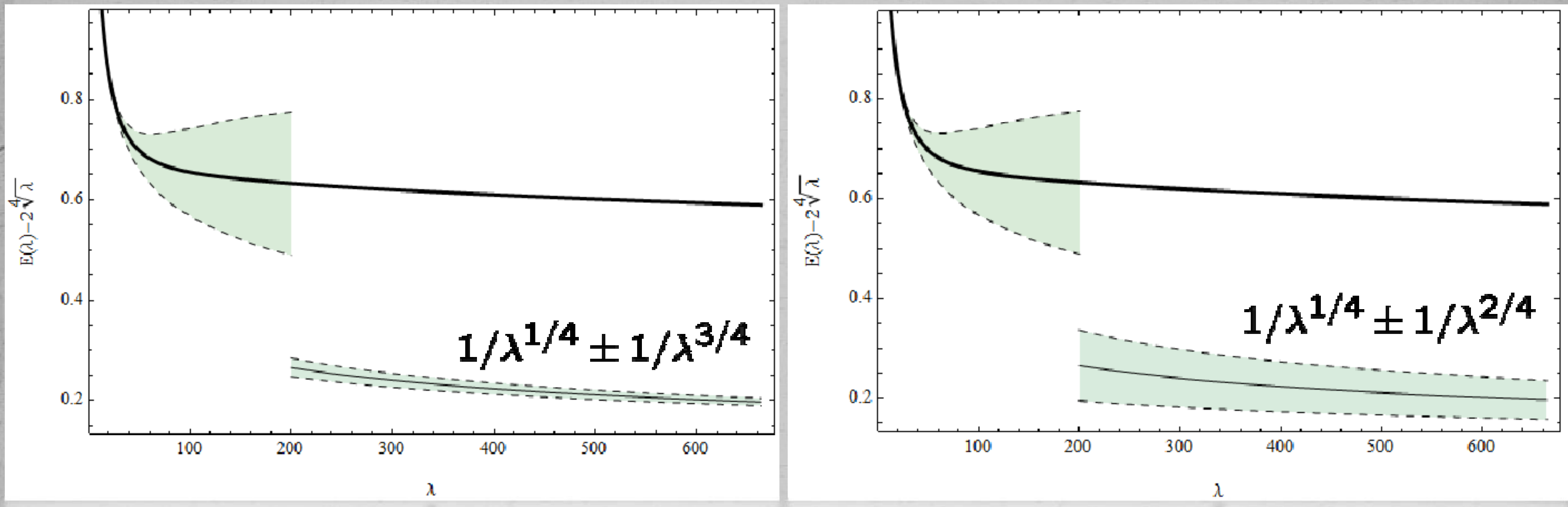
Numerical results



- 20 computers 4 nights
- Emergence of integers – a miracle
- Exact BAE – modified by phase

Porto university
Wolfram

Comparison with Roiban and Tseytlin



Conclutions

- Y-system is proven to be a computational tool
- We reproduced several leading terms at strong coupling for a simple operator
- Any sl2 operator could be computed readily for any g
- Generalization to other sectors to be done
- Analytical results – coming soon?
- Other AdS/CFT's?

N.G., Kazakov, Vieira